PHOTOMETRY OF Lights of Different Colors

HERBERT E. IVES.

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Studies in the Photometry of Lights of Different Colours.

By HERBERT E. IVES.

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INTRODUCTION.

ANY light may be defined in terms of hue, saturation, and luminosity. Practical photometry is concerned primarily with luminosity. But that quality of light cannot by simple means be separated from the other two. The eye can equate but cannot appraise. Hence the eye is on uncertain ground whenever it attempts to compare the luminosities of lights of different hue or saturation. This difficulty has long been recognized, and various opinions as to the solvability of the problem have been expressed. Various plans to accomplish its solution have been proposed. The belief has been expressed by some that comparison of the luminosities of two differently coloured lights is no more possible of exact accomplishment than the exact comparison of a sound with an odour. Helmholtz declared his lack of confidence in his own ability to make such comparisons. On the other hand, some observers have by practice acquired skill and apparent certainty in heterochromatic photometry. Again, methods of photometry have been discovered and developed which, in the process of measurement, evade and perhaps eliminate the differences in hue and saturation.

By these different methods different criteria for equality of luminosity are obtained. What are the relations between the criteria? Are two coloured lights which are equally bright as measured by one method, of equal brightness as measured by another?

The relative brightness of differently coloured lights is not constant under all conditions. Physiological factors enter to a large degree. The "Purkinje effect" is the name given to the increase of blue sensitiveness at low illumination. The "yellow spot effect" is the name given to the change in the relative brightnesses of different colours occurring when the size of the field of view is changed. To what degree do these enter in the various methods of colour photometry?

The advent of high efficiency incandescent lamps and of new illuminants with wide range of colour, the prospect of an even greater variety of illuminants in the future, diverging still further in colour from the present flame and carbon lamp standards, all render the problem of measuring the luminosity of lights of various hues one of the most important in photometry. A complete solution of the problem can come only through knowledge of the relative behaviour of each method of comparison under the various conditions known to affect the relative brightness of different colours. That knowledge acquired, we may proceed to a process of test and selection, and so hope to determine that method which meets to the fullest degree the requirements of a method of measurement.

The chief obstacle in the way of giving preference to one method of colour photometry over another is at present our lack of data. Some forms of flicker photometers have been compared with some forms of direct comparison photometers. Spectral luminosity curves for the eye have been derived by tests of visual acuity, by the flicker photometer, by equality of brightness methods, and by the determination of critical frequency of flicker. But such investigations have been carried out by different observers, using different conditions of illumination and different colour standards, with instruments whose dimensions were different in essential particulars. And frequently some of the most important conditions have not been stated. Prominent among these should be mentioned the illumination and the size of the photometric field.

It is the object of the present studies to obtain data for the solution of this problem. For this purpose comparisons will be made between the various methods. The different factors which influence the relative luminosity of colour will be systematically varied as each method is studied. The same conditions will, however, be maintained throughout, whereby satisfactory comparisons of the different methods will be possible. Attention will be devoted chiefly to the most promising lines of investigation, and ground covered by previous workers will not be touched unless coordination of their work with the present work is found impossible. Problems incidental to the main questions will be investigated where necessary, but the ultimate object to be held in view will be the determination of the most satisfactory method of measuring luminosity where differences of colour exist.

Existing Methods of Heterochromatic Photometry.

The chief methods of light measurement which have been applied to differently coloured lights are :—

1. *The method of equality of brightness.*—The two lights illuminate the two parts of a photometric field. The relative intensities of illumination from the two lights are changed until the two differently coloured fields give a sensation of equal brightness. The illuminations are then called equal.

2. *The method of visual acuity.*—Two illuminations are equal when the same fineness of detail is just distinguishable by each.

3. *By critical frequency.*—Two illuminations are equal when the flicker produced by rapid alternation of one illumination with black disappears at the same speed of alternation as for the other when similarly alternated.

4. *The flicker photometer.*—Two illuminations are equal when upon rapidly alternating one with the other no sensation of flicker results, the speed of alternation being such that the slightest change of either illumination produces a flicker.

The Requirements of a Good Method of Measurement.

As a preliminary to an investigation of these methods of measurement, the requirements necessary to a satisfactory method should be kept clearly in mind. These are, first, that the range of possible settings of the measuring instrument shall be small; that is, the method shall be as definite as possible. In the photometry of lights of the same colour, for instance. the range with a Lummer-Brodhun contrast photometer is of the order of one per cent. Second, that the quantities to which values are given by the measurements shall behave as arithmetical or physical quantities. This involves that quantities measured as equal to the same thing shall measure equal to each other, and that the sum of the parts shall be equal to the whole. This second requirement of a method of measurement is not one often dwelt upon in the measurement of purely physical quantities, because such quantities naturally fulfil the requirement. In dealing with physiological quantities, however, it cannot be assumed without proof that this is true. As a third requirement in the present case may be added that the values given to different colours shall represent as nearly as possible their useful value as light. In explanation of this last requirement it will suffice to point out that the first two requirements, which pertain to any method of measurement, would be met by simple measurement of the energy radiated. Light, however, considered as a sensation, is represented by very different quantities of energy at different wave-lengths in the visible and invisible spectrum, and such a measurement would not give a number to the light value. Or, again, if there exists a fundamental quantity of brightness, which can only be separated and measured under very special conditions, this measurement, while perhaps satisfactory to the psychologist or physiologist, would not satisfy the photometrist, who is interested in light as used for illumination, even if the underlying real brightness quality is masked by other qualities such as hue. What is therefore required is a method of measuring lights of different colours with the same order of definiteness which pertains to strictly physical measurements, using units which may be dealt with as physical quantities, but yet yielding values closely representing the values of the lights for illumination, for good seeing, or as producers of the sensation of brightness.

Degree to which these Requirements are met by Existing Methods.

Comparison of the brightness of two fields of different colour is a somewhat uncertain and unsatisfactory process.

The difficulty is largely psychological. A decision must be formed as to when a certain quality of the two illuminations is equal, while other inseparable qualities are different. With a large colour difference, this decision is difficult; at first attempt almost impossible. At best, one's settings are apt to vary over a wide range; at another time a different decision may be made, and the average of a set of readings diverge considerably from the set previously made. With small colour differences an observer will with practice make quite definite settings, and his criterion of equality then probably assumes a fairly constant value. As a method of heterochromatic photometry, the method of direct comparison is principally open to objection because of its lack of definiteness of setting.

As to the second requirement, that the measured light quantities may be treated as physical quantities, *i. e.*, compared in any order or added to produce their arithmetical sum, Abney with his colour-patch apparatus has measured the intensities of several spectral colours separately and then together, finding practically perfect agreement between the sum of the intensities and the intensity of the sum. In view of the difficulty of making exact settings with coloured fields, those results show remarkable agreement. It is to be regretted, however, that these measurements were made with such an inexact instrument as a shadow photometer, that the illumination from the different colours (an arc spectrum) was very different, and that the measurements were confined to very small fields: in short, that they were made under conditions quite different from ordinary photometry.

As to the method of visual acuity, its chief defect is extreme lack of definiteness. Properly it should be used only for determining different orders of illumination. Visual acuity varies as the logarithm of the illumination and very slowly. For instance, with the best test objects probably a 5 per cent difference in visual acuity represents the limit of perceptible difference. Study of Kœnig's data on the relation between visual acuity and illumination shows that this corresponds in the middle region of illumination studied to about 30 per cent. difference in illumination. As a method of measuring illumination, visual acuity is therefore almost beneath notice. This is borne out by the work of all observers; so that while visual acuity may be a correct measure of good seeing conditions, it is not a method of photometry. Until, therefore, it shall be proved that the criterion of good seeing as so determined is very different from that given by other methods, and much more important, visual acuity would not appear to deserve much attention. To the writer's knowledge no measurements are on record which were made to determine whether the arithmetic sum of two differently coloured illuminations measures equal to the two illuminations together. One advantage the method of visual acuity has over that of equality of brightness, is that by the nature of the criterion two illuminations equal to the same are assuredly equal to each other. This fact depends of course on the illuminations not being compared one with another, but with, as it were, an outside standard of different character. The importance of this will depend on whether in other methods any error is shown due to characteristics of the compared lights, other than their luminosity.

Measurement of illumination is possible by determining the critical frequency of flicker. The method is subject to very much the same criticisms as apply to measurement by visual acuity. Taking as illustration the investigation of Kennelly and Whiting, we find that " every time the illumination on the target is doubled, the mean vanishing flicker frequency increases by 3·3 cycles per second approximately." The observations of each observer used to obtain the mean of three show sometimes values $1\frac{1}{2}$ or 2 cycles per second above the mean adopted, sometimes as much below, indicating a possible range in settings corresponding to about 100 per cent. in illumination. In short, the method appears to be not so much for accurate light measurement as for orders of illumination. It possesses the characteristic noted in connexion with visual acuity methods, namely, that it is not necessary to view the two differently coloured fields simultaneously in order to make a measurement; two illuminations equal to a third are, therefore, of necessity equal to each other. Tests of the results of adding illuminations of different colours are lacking.

A word may be said in passing in regard to the common characteristic of the last two methods, that by neither is it necessary to juxtapose the observed illuminated surfaces. In one way this is an advantage. The problem of judging between two hues is eliminated. Judgment is made on a quality common to both illuminations (their detail or flicker revealing power) which is not confused by hue, as is the sensation of brightness. On the other hand, both visual acuity and ability to perceive flicker are largely influenced by physiological conditions such as adaptation and fatigue, and are apt to vary from time to time. The uncertainties introduced by these latter are probably as great as the uncertainties of individual settings during one measurement. At any rate it is significant that none who have worked with either visual acuity or critical frequency in their relation to illumination have seriously proposed using them as means of exact photometry.

The last method to be considered is that of flicker photometry. The flicker photometer owes its applicability to heterochromatic photometry to the fact that colour fusion occurs before brightness fusion; that is, the sensation of flicker caused by alternation of one colour with another disappears at slow speeds of alternation, resulting in a field of uniform hue in which is an outstanding flicker. This outstanding flicker disappears only when the two illuminations bear a certain relation to each other. For lights of identical colour the point of disappearance of flicker corresponds exactly with the point of equal illumination. With lights of different colour this point certainly corresponds closely with equality as measured by other methods, though perhaps not exactly.

The flicker method has found many advocates because of its superior definiteness as compared with the method of direct comparison. The comparison of different hues is eliminated, apparently by a process which separates luminosity from hue, leaving a phenomenon—flicker—of considerable delicacy. Whitman, Rice, and others find settings between lights of different colours nearly as easy as between lights of the same colour. The accuracy of setting compares well with that of direct comparison between lights of the same colour.

Work by Whitman and Tufts would appear to show that the flicker method possesses this further requirement of a method of measurement: that several illuminations acting together measure to the arithmetic sum of their values measured alone. In short, the flicker method appears to yield greater definiteness than the method of equality of brightness, and to have the same qualifications of a method of measurement as Abney's work would show for the latter. Spectral luminosity curves obtained by the flicker photometer have shown considerable similarity to those by other methods, so that the special requirements of a method of coloured light photometry appear to be well met.

From this review it seems that the study of methods of heterochromatic photometry would most profitably centre around the equality of brightness and flicker methods. Concensus of practical opinion agrees with this, for it is with the different values given by these two methods that photometrists are now largely concerned. From the manner in which each meets the requirements outlined above, it might seem as if both are suitable, with the advantage in favour of the flicker method because of its greater definiteness. It might be expected too that the methods would agree. Yet apparently this last is not the case. The flicker method has not been so generally used as its convenience would recommend, because numerous disagreements have been recorded between its renderings and those of the longer established method of direct comparison or equality of brightness. The explanation of these differences is most likely found in the different conditions which have obtained when the characteristics of the two methods have been studied, and when comparisons have been made between them. The importance of certain physiological conditions will be emphasized in the next section. The most significant facts are that the methods do not in general agree, and that the tests to which they have been subjected, in particular by Abney and by Tufts, were under conditions radically different than those of practical photometry. The most pressing subject for investigation just now is, therefore, the relation between these two methods, under various conditions. What the important conditions are will be made clear by consideration of certain physiological factors.

Physiological Factors.

The relative sensibility of the eye to different coloured lights is considerably affected by changes in illumination and by changes in the size of the field of view. According to present theory, these differences are due to the irregular structure of the retina and to the preponderance of different seeing elements at different illuminations. Two sets of elements are present in the retina—rods and cones. According to the " Duplicitäts Theorie " of von Kries, the rods are chiefly responsible for vision at low intensities, the cones at high. The cones are sensitive to colour with a maximum of sensibility in the yellow ; the rods are not sensitive to colour, but have a maximum of sensibility at a wave-length corresponding to the green. These elements are distributed unevenly over the retina. At the centre (the fovea) there are no rods; both rods and cones are about equally distributed in the surrounding central region called the *macula lutea;* while in the outer regions of the retina rods predominate. Hence, either increasing the size of field or decreasing the illumination brings into play more of the rods, thereby explaining the Purkinje and yellow-spot effects.

Whether the rod and cone theory will prove the true explanation of retinal action is not at present of great moment. The facts about retinal structure on which it is based are, however, to be kept in mind while studying colour photometry, as indicating the most probable critical points in the series of changes which may be rung on conditions.

The complications introduced by colour blindness will not be a main topic of the present research. The object will be to secure data from normal eyes, on the understanding that only observers of normal vision are equipped for photometry with lights of different colours. The comparison of various methods with one another, and the investigation of each as to its qualifications as a method of measurement, can be made by one observer, and in fact should be so made first of all, as thereby differences in method are not confused with differences between observers. Some data obtained by several observers of normal vision have however been collected with a view to testing the generality of the phenomena found, and to determine how much variation exists among observers with no marked abnormalities of vision.

Adaptation and fatigue are probably the physiological factors most difficult to estimate and control. Colour vision is peculiarly dependent on adaptation. When physiologists work on problems of vision they distinguish between the light-adapted eye and the dark-adapted eye, depending on whether the eye works in the light or in comparative darkness. The ability to perceive colour, which is lost on greatly decreasing the illumination, gradually returns as the eye becomes accustomed to the small quantity of light. Fatiguing the eye for one colour makes it more sensitive to others. Fatigue also alters the relative critical frequencies of flicker for different colours ; and the effect is different depending on the character of the fatiguing light.

Both these disturbing factors must be kept in mind when investigating heterochromatic photometry, but their detailed investigation must wait until the more prominent phenomena have been covered. It is to be noted that the effects of adaptation and fatigue are brought out by extreme conditions, such as very high or very low illumination. In ordinary photometry the eye is probably in the condition called by physiologists "light adapted," because of the order of illumination used, and because of the use of auxiliary lights for such things as scales and data sheets. If observation has not been too long continued or if the illumination is not trying, the effects of fatigue may be kept at a minimum. The policy in the present investigation has been to maintain the conditions of lighting at nearly those of practical photometry, and, by making observations for only a limited time each day, to prevent the eye from becoming fatigued to such an extent as to be noticed by difficulty or discomfort in making readings.

Summarizing the physiological considerations we find that an investigation of heterochromatic photometry must include studies of the effect of varying the size of the photometric field and of varying the illumination. A complete study will include observations for a large range of illuminations and for several sizes of field at each illumination. The state of adaptation of the eye must be as nearly as possible that holding under the conditions of measurement to which results are to be applied.

I.

Spectral Luminosity Curves as obtained by Flicker and Equality of Brightness Photometers.

Previous comparisons of the two photometers.—The most extensive comparisons heretofore made between these photometers are those of Dow and Stuhr. Comparisons of less complete character have been made by Millar, Wilde, and others. Dow, recognizing the importance of the physiological factors above emphasized, has studied the effects of change of illumination and of field-size. For this purpose he used coloured glasses over his light sources. Comparing, for instance, red and green lights, he found by direct comparison that on decreasing the illumination the relative brightness of the red decreased, the decrease being very rapid from ·2 metre candle down. With the flicker method this decrease was hardly observable, and the change at ·2 metre candle was not shown, although the difficulty of flicker measurements is too great at low illuminations to establish this difference with certainty. On decreasing the size of the field, at a fixed illumination, the brightness of the red increased by equality of brightness; by the flicker method a much smaller increase was found. These experiments were carried out with the same sized field and the same illumination for each method, so that on that score the results are beyond criticism. They indicate that the flicker method is less susceptible to the influence of changed illumination or size of field.

Stuhr has compared the methods of visual acuity, direct comparison, critical frequency, and flicker, using coloured glasses. He found the methods by critical frequency and by flicker to yield identical results. These differed both from the results of visual acuity and equality of brightness measurements, lying between these latter. Different field-sizes —unspecified—were used for visual acuity and other measurements, and in no case was attention paid to the absolute illumination. In other words, the physiological factors were not considered.

P. S. Millar describes measurements of a mercury arc against incandescent lamps in which the illumination varied over a wide range. With equality of brightness photometers the mercury arc measured relatively much brighter at low illuminations : with flicker photometers this effect was absent.

Wilde describes measurements of a tungsten lamp against a carbon, made with a flicker photometer and with an equality of brightness photometer. The photometers showed a disagreement of several per cent.

Of these comparisons that of Dow is by far the most complete and scientific, establishing as it does that the two methods give different results for the same size field when the illumination is changed, and that at a fixed illumination they respond differently to changes of field-size. The two points in which Dow's work is incomplete hold also for the work of the other investigators quoted, while their work is less complete in other ways. First, it is to be noted that the experiments refer to no definite scale of colour, such, for instance, as the spectrum. It is therefore impossible to calculate from his results the magnitude of the effects of varying conditions for any given energy distribution. Second, while the effect of varying either illumination or field-size was investigated, the investigation was not continued to the point of varying both together. There is, for instance, a large range of illuminations suitable for the carrying out of measurements. At each of these illuminations any one of several sizes of field may be used. Our knowledge of the relative behaviour of the two methods is incomplete until we know, on some definite colour scale, how these two variables—illumination and field-size—affect the criteria furnished by the methods, for all practicable combinations of the variables. The importance of this knowledge may be indicated by considering the character of the differences between the methods as indicated by Dow's work :—

The Purkinje effect is less with the flicker method. We also know from the work of Kœnig and Brodhun that the Purkinje phenomenon practically disappears for high illuminations. The effect of decreasing field-size is (according to Dow) less for the flicker method, but we also know that the Purkinje effect by direct comparison has been found much less for small fields. That is, by using high illuminations and small fields the differences between the methods are less than for low illuminations and large fields. At once there opens the possibility that by proper choice of illumination and field-size the two methods may agree. Should this be the case, it would be a strong argument for choosing that illumination and field-size as the standard one for making colour comparisons. The possibility thus indicated is sufficient for undertaking a complete comparison of the methods.

Apparatus and Procedure.

The method of attacking the problem was to determine by the two methods and under various conditions the luminosity curves of a spectrum of known energy distribution. The apparatus used is similar in some respects to that of Tufts, and is shown in fig. 1. A Hilger constant-deviation

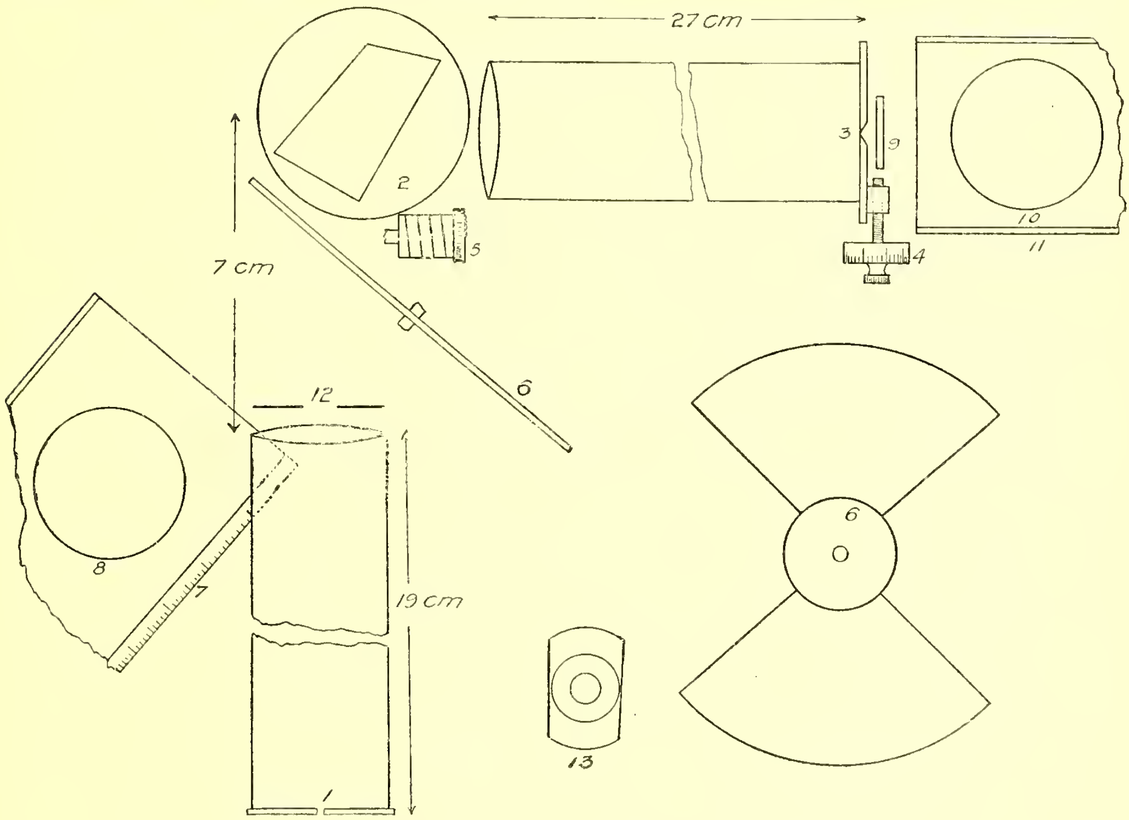

Fig. 1.—Arrangement of Apparatus.

1. Observing slit.	6. White sector-disk.	10. Light source.
2. Prism table.	7. Photometer bench.	11. Photometer bench.
3. Collimator slit.	8. Standard lamp.	12. Diaphragm.
4. Divided drum.	9. Diffusing glass.	13. Diaphragms used.
5. Wave-length drum.		

wave-length spectrometer forms a spectrum of the light source upon an eye-slit (1) of dimensions $\frac{1}{2}$ by 2 mm. The eye placed at this slit observes the prism face (2) illuminated by monochromatic light of a purity determined by the relative size of collimator and eye-slit (3 and 1). The wave-length incident on the eye-slit is changed by motion of the drum (5). The size of the collimator slit is changed by turning the drum (4) which is provided with a divided head. At (6) is a thin metal disk of the shape shown in separate sketch. This has carefully bevelled edges and is painted white. Periodically the disk is smoked over burning magnesium wire, so that the surface used is of white magnesium oxide. At (7) is a photometer track, carrying a movable incandescent lamp (8). The disk is connected by a belt with a direct-current series-wound motor in series with a variable resistance, and the same belt passes over the pulley of a Weston electric tachometer. At (9) is a piece of finely ground glass; at (10) a frosted bulb tungsten lamp, movable on a photometer track (11). At (12) may be placed diaphragms of any desired size.

For light source at (10) the requirements to be met were: first, as high intensity as possible; second, as white a source as possible, since prismatic dispersion and the selective absorption of the prism glass both decrease the amount of blue light for practicable slit-widths. A tungsten lamp burning at normal voltage was ultimately decided upon. At frequent intervals this was checked for colour upon a photometer by ascertaining the voltage to give the integral colour of three standards which were not burned at any other time. This proved fairly satisfactory, although some change in distribution undoubtedly occurred during the investigation, due partly to blackening of the bulb.

For light source at (8) carbon lamps were used, of uniform colour, ranging in candle-power from 65 to 5. These placed at various positions along the photometer bench permitted variations of illumination from 1 to 600 metre candles. The colour chosen was that of the incandescent lamp secondary standards at present in use, namely that of a 4·85 watts per mean spherical candle treated carbon, oval anchored filament. Use of an unsaturated colour of a particular hue will of course give results applying only to comparisons with such a standard. The complete study of the question will probably demand investigation of any possible effects of altering the character of the standard light.

The method of obtaining the luminosity curves is as follows:—The lamp (8) is placed to give a certain illumination; the disk (6) is turned so that the photometric field is bisected; the drum (5) is set at a point near the middle of the spectrum (taken always arbitrarily to give a mean wave-length of ·574μ); the divided head (4) is turned to give a slit-opening of 10 units (0·1 mm.): the lamp (10) is then moved until the two halves of the field are, as nearly as can be estimated, equally bright, at which point the lamp is fixed. A set of readings is then made, first by equality of brightness, then, with the disk rotating, by the flicker method, intensities being matched by changing the width of the slit (3). The wave-length drum is then changed to a point in the red, next to a point in the green or blue; and so alternating from one side of the original point to the other, similar double sets are made. When the whole spectrum as far as practicable is covered, a set of slit-widths is obtained, the reciprocals of which are as the brightnesses of the colours. By this means the illumination is constant throughout any one measurement—an important condition, which has been overlooked by several investigators.

Three sizes of field are used: one corresponding to the whole prism-face, an oblong 18 × 24 mm., somewhat larger, as viewed at 20 cm., than the angle subtended by the macula lutea; the second, a circular opening of 16 mm. diameter, subtending at a distance of 20 cm. an angle a little less than the macula lutea; the third, a circular opening of 6·5 mm. diameter, corresponding to the angular size of the fovea. At each illumination a set of equality of brightness and flicker measurements is made for each field-size. The making of such a set for one field-size consumes a morning or afternoon, and but one can be made safely in a day without unduly fatiguing the observer.

The measurements so obtained are subject to several corrections. These are: first, that for slit-width; second, that for prismatic dispersion; third, for the selective absorption of the prism; fourth, for the energy distribution of the source whose spectrum is used. The illumination as measured by the intensity and distance of the lamp (8) must be corrected for the loss by absorption through the telescope lens.

To determine the slit-width correction it is necessary to know the approximate luminosity curve and the portion of the spectrum included in each measurement. The latter was determined for the whole range of slit-widths used, and for the largest slit was found to amount to ·03 μ at ·66 μ, and to ·01 μ at ·49 μ. In the yellow, where the luminosity curve changes its direction most rapidly, the greatest width used is ·016 μ. In this region the correction calculated from the approximate luminosity curves is 0·6 of one per cent. This, the largest correction, is of the order of magnitude of the errors of observation, and may therefore be neglected.

The correction for prismatic dispersion was determined by measurements of the wave-length interval corresponding to the width of the eye-slit, using the drum divisions and spectral lines of known wave-length.

The correction for prism absorption and for energy distribution were determined together. Spectrophotometric comparison was made between the light of an incandescent lamp previously compared with a black body of known temperature, and the light of the tungsten lamp source through the ground glass (3) and the prism (2). Employing the Wien equation $J_\lambda = c_1\lambda^{-5}e^{-\frac{14500}{\lambda T}}$ to determine the energy distribution for the black body at the temperature measured, the distribution of energy from the source as used is given in fig. 11 (Pl. III.).

The calculated illuminations due to the various lamps placed on the photometric bench (7) are subject to a reduction of 10·5 per cent. as determined by measuring the absorption of the telescope lenses in a photometer.

A large number of observations of the character described were made almost daily during a period of about six months by the writer, and by other observers as will be noted. Illuminations were used whose corrected values are 1·8, 2·9, 8·9, 16·1, 26·9, 33, 68, 110, 175, 270 metre candles. These illuminations, because of the artificial pupillary aperture of 1 sq. mm., correspond to lower illuminations in practical photometry; they will therefore be referred to here as "illumination units."

The observations were plotted on thin, rather transparent coordinate paper, which permitted each observation to be compared with any other by simply superposing and holding to the light. This plan suited admirably; for in order to make all the comparisons necessary, each curve had to be studied in several different combinations. The same plan is of course impossible in publication, so that it will be necessary to select certain representative curves and to plot several in each diagram; the same curves will for that reason figure several times. The necessity for this will be evident from the following list of the combinations necessary for a complete comparison with one observer alone.

(1) *Effect of changing illumination, for each of three field-sizes.*

- (*a*) On the direct comparison curves.
- (*b*) On the flicker curves.
- (*c*) Relative position of the two curves due to the change.

(2) *Effect of changing size of field, at each of ten illuminations.*

(*a*) On the direct comparison curves.
(*b*) On the flicker curve.
(*c*) The relative position of the two curves due to the change.

Among the curves are some showing bad points and some which, as the text indicates, do not exactly agree with those taken at other times. These imperfect curves (practically all "equality of brightness" curves) are used because even with their defects they show a proper relation to the preceding and following ones, while curves made at a later date purposely to replace them would correspond to a changed criterion. Attention must be called to the fact that in the measurements involving physiological variables, it is impossible to exactly reproduce conditions holding at one time; hence data must occasionally be used for which, were it possible, one would by preference substitute other determinations. The curves selected have been most carefully chosen so as to represent most clearly the various phenomena brought out by the measurements.

Every effect to which attention is called and every conclusion is supported not only by the curves shown in this article, but also by others made at intermediate illuminations. Further, to avoid any possibility of error due to change in the light source between measurements, check settings at one time have been made to verify (at least qualitatively) those phenomena which are indicated by the differences between curves made on various days.

The data here presented consist principally of three sets:—

1st. Measurements made by the writer, chiefly during a period of about eight weeks in May, June, and July, 1910.

2nd. Measurements made by Mr. Luckiesh during July and August.

3rd. Measurements made at selected points for one field-size by five observers of normal colour vision during two weeks in September.

These will now be considered from various points of view. In turn will be taken up:

(*a*) Relative sensibility of the flicker and equality of brightness methods.
(*b*) Reproducibility of measurements.
(*c*) Effect of changing illumination on each method for each field-size.

(*d*) Effect of changing field-size on each method, for various illuminations.

(*e*) The relative positions of the luminosity curves by the two methods at various illuminations and field-sizes.

(*f*) Comparison of luminosity curves obtained by different observers.

(*a*) *Sensibility.*

A measure of the relative sensibility of the two methods is obtained by taking the mean error of setting. In fig. 2 (Pl. III.) are plotted the mean errors for five observers at 250 illumination units for various points in the spectrum, from the deep red at ·66 μ to the green at ·51 μ. The mean flicker errors are given by the full line, the mean equality of brightness errors by the dashed line. In fig. 3 are given the same quantities as obtained from observations at 10 illumination units. Several facts are here brought out clearly. First, the flicker method is for all parts of the spectrum several times as sensitive as the equality of brightness method at the higher illumination, the relative sensibility differing for different observers, but always favouring the flicker method. Second, the difference in sensibility between the methods is greater at high illuminations than at low. At the lower illumination the equality of brightness sensibility becomes greater, the flicker sensibility less. Third, the sensibility by the flicker method is less toward the ends of the spectrum, where the difference in hue between the spectrum colour and the standard lamp colour is greatest.

It is to be noted that while all five observers made readings by the flicker method which compared very closely to the accuracy of those made by the two most experienced observers (H.E.I. and M.L.) two of the others (by whom settings were made for the first time) averaged five or ten times the error by the equality of brightness method as by the other. This illustrates the great superiority of the flicker method as a method of measurement with observers not used to making matches between lights of widely different colour.

In connexion with the question of sensibility may be given the data on the speeds used with the flicker photometer. As is well known, sensibility varies with the speed, being less at high speeds. In fact, with high enough speed all flicker vanishes, no matter what the difference in illumination from the two sources under comparison. In these experiments the speed was always adjusted to the lowest value at which flicker could be made to disappear. This was then read by the tachometer and reduced to cycles per second. In fig. 4 (Pl. III.) are given the speeds as used by the different observers from end to end of the spectrum and for the two illuminations. The data show that much lower speed is necessary for low illumination, and that the ends of the spectrum require higher speed than the middle. These facts readily fit in with our knowledge derived from other sources and with such theory as we have of the action of the flicker photometer. The eye is more sensitive to flicker at high illumination than at low, hence the greater sensitiveness of the flicker method and the higher speed necessary at high illuminations. In order to compare lights of different colours it is necessary to attain such a speed that the colour flicker, due to difference in hue, disappears. It is therefore to be expected that at the ends of the spectrum where the hue is most different from the comparison lamp, a higher speed is necessary, and with this higher speed goes decreased sensibility. Whether the change in sensibility is exactly what the change in speed would occasion, or whether the change in speed is conditioned by hue difference alone, are points for future study.

In fig. 5 (Pl. III.) are plotted some preliminary data upon flicker speeds and sensibility under varied conditions of field-size and direction of vision. The two upper curves show the speeds (at 270 illumination units) for the largest and smallest fields used. The large field requires the higher speeds ; in other words, it is more sensitive to flicker. The lowest curve gives the speeds used for observation by peripheral or averted vision. To obtain these the smallest aperture was displaced to one end of the total available opening, and attention was fixed upon a small pin-hole (in which no flicker was visible) at the other end, about 8° distance, flicker being observed at the small aperture. The most striking feature is the much lower speed, corresponding to that required for lower illumination. Above are plotted the mean errors for observation by peripheral vision. These are much larger than by central vision. Later, the theoretical bearing of these points will be discussed. It is hoped ultimately to follow them up with more extended investigations of the retinal field, not only for speed and sensibility, but also for the character of the luminosity curves. These have, however, been deferred until some of the points of more direct interest to photometry are studied, although the complete explanation of the phenomena encountered will probably be brought out by study of extreme conditions, snch as those holding in peripheral vision.

(*b*) *Reproducibility of Measurements.*

Early in the course of the investigation a brief test of the reproducibility of measurements was made. It was then found that measurements made from day to day agreed to within the errors of measurement; that is, the individual flicker points agreed to within one to five per cent. depending on the part of the spectrum, while the equality of brightness points agreed somewhat less well. The chief object of the investigation at that time being to obtain knowledge of the relative positions of the two kinds of luminosity curves as simultaneously obtained, no further study of the question of reproducibility was made until the great body of curves here plotted were obtained.

At the conclusion of the measurements by the second observer (M.L.) the writer made a number of check measurements, which established a fact that had before been indicated by other occasional measurements, and by the measurements of M.L. This is that the equality of brightness curve may change its position with respect to the flicker curve in course of time. It appears that one's idea or criterion of a match in intensity by equality of brightness may change as compared with the flicker criterion, assuming the latter to remain fixed. Thus in the set of measurements given in Plate I. (Pl. IV.) the equality of brightness method showed, for large fields, the red less bright in proportion to the blue than did the flicker method. Three months later, after an interval of other work, the writer obtained, under the same conditions as before, curves with the red brighter by the equality of brightness method than by the flicker method; in other words, the equality of brightness curve had shifted from one side to the other of the flicker curve. The first curves of Plate I. and Plate V. (made on different lamps) show this change of relative position just described.

As to the amount of change experienced by each curve, the data as obtained do not give as complete an answer as is desirable, because some changes in the distribution of energy in the tungsten lamp took place in the interval of time, due to blackening, etc.; and in the last measurements an entirely new lamp had to be used. As the curves stand, they show slight changes in the flicker points, but very large in the equality of brightness ones. Taking into account the changes and differences in the lamps, it appears extremely probable that the flicker luminosity curve remains fixed to within the errors of measurement, while the equality of brightness curve remains fixed to within the errors of measurement only so long as one's memory retains the idea first formed as to the brightness of coloured fields. In course of time the latter curve may experience changes much greater than the change given by the errors of measurement.

A satisfactory test of reproducibility can only be made with lamps and apparatus standing absolutely unchanged between measurements—a condition which cannot be absolutely maintained. Nevertheless, a test approaching this in character has been carried out since the conclusion of the comparative measurements by different observers. Two seasoned "4-watt" carbon lamps have been used as sources, the apparatus has been clamped rigidly in position, and at intervals of about a week, for a period of two months, flicker and equality of brightness curves have been made by two observers. The middle-sized field was used, at the highest illumination (270 units).

In fig. 6 (Pl. III.) are plotted the mean deviations from the mean, or the mean error of setting, where the mean of all is taken as correct. These show less satisfactory reproducibility by the equality of brightness method than by the flicker method. The difference is not everywhere, however, as great as might be expected from the relative sensibility data and the considerations above mentioned. It must, however, be borne in mind that these results are obtained by two very practised observers, whose criterion of equality of brightness had become well fixed long before those measurements were made. With new observers or with observers who rarely made comparisons involving such large colour differences, the equality of brightness method would not have shown up so well. The indications are that with practice one's criterion becomes apparently fixed so that equality of brightness comparisons can be made with a high degree of reproducibility, though not with as high a degree as the flicker method. Whether this criterion is correct or not is another matter.

A point which deserves mention is the question of reproducibility at low illuminations. There it has been found by experience that reproducibility is much less satisfactory by both methods, and for a pretty clear reason, viz. one's illumination scale at low illumination is not a fixed but a shifting thing, due probably to the different physiological condition of the observer at different times. A set of curves for three field-sizes, for instance, will be obtained on three successive days and will show a certain relation at a given low illumination. After an interval of several days or any interruption of habits, an attempt to check these may give three curves consistent with themselves but corresponding either to a higher or lower illumination than the earlier curves. It is thus out of the question to ascribe a certain luminosity curve to a definite illumination, since the same illumination in metre candles corresponds at different times to different retinal conditions. Where then in the appended curves the illuminations are given for the low illumination observations, the figures indicate under what conditions the measurements were made, but do not necessarily mean that any one curve is only to be obtained at just that number of metre candles. The order of illumination is the important thing here, and the trend of the phenomena with changing illumination.

Work on the border-line between physiology and physics tends to shake one's faith in the principle of universal causation. Faith may be reconciled with experience only by constant reminder that however exactly physical conditions may be reproduced, one's physiological apparatus of seeing cannot be brought to a previous condition by any amount of careful work with metre stick or voltmeter.

These remarks on the shift of one's illumination scale apply only to illuminations of the order of twenty units and below; above that figure this difficulty disappears, and for that reason the reproducibility test described above was made at high illumination. Needless to say, the difficulties of low illumination measurements have necessitated close application and considerable repetition of work in order to secure the data presented.

(c) *Effect of changing illumination, on each method, for each field-size.*

In Plates I. and I. *a* (Pl. IV.) are given luminosity curves as obtained by the two observers by whom the complete sets of measurements (illumination and field-size varied) were made. These as plotted are corrected for prismatic dispersion, but not for the energy distribution of the source, and are arbitrarily made equal at $\cdot 574\,\mu$. In the upper row are shown the curves obtained by the flicker method, four different illuminations being plotted together, in three groups, each representing one size of field. In the lower row the same data are given for the equality of brightness method. Appended keys indicate the illumination and field sizes in question.

From these curves it appears that the effect of change of illumination is to shift the luminosity curves along the spectrum. The most striking phenomenon is that while the shift with the equality of brightness method is toward the blue (Purkinje effect), the shift with the flicker method is toward the red, or exactly opposite. Also to be noted is that with the smallest field these shifts are less marked.

Examining the curves more in detail, several interesting points appear. Taking first Plate I. (H.E.I.) and examining the quality of brightness (lower) curves, it appears that the effect of decreasing illumination is two-fold: first occurs a broadening of the curves (increase of area), then an increase on the blue side, which results in a shift of the whole curve toward the blue. This may be interpreted as an increase on the red side, followed by an increase on the blue side, the latter becoming more marked at lower illuminations. With decrease in the size of the aperture these changes are much less, which is in accordance with the oft encountered statement that the Purkinje effect is absent for very small fields.

The flicker curves exhibit a plain shift toward the red, with not much difference for the different field-sizes until the lowest illumination is reached. At this lowest illumination there appears a drop on the red side, most marked for the largest field-size. In consequence of this the area of the curve is less and its maximum shifts again toward the blue.

Consulting Plate I.*a* we find the same general phenomena exhibited, a shift toward the blue for equality of brightness, toward the red for flicker, with decrease in illumination. Some differences between the results of the two observers are to be noted. Chief among these is that the corresponding changes in the curves take place at different illuminations and field-sizes from those of the first observer. The lowest measurement, for instance, was made at an illumination where the writer could secure no reliable flicker measurements.

As to the equality of brightness measurements, it is to be noted that the increase of luminosity on the red side of the curve is absent with the large aperture, slight with the middle aperture, and pronounced with the smallest aperture. The smaller increase of the blue sensitiveness or Purkinje effect with the small field is not only present, but because of the enhanced red sensitiveness the maximum of sensibility for low illuminations actually shifts toward the red, somewhat as the flicker curve for the same field-size and illumination. As a consequence of these facts the curves of this plate do not show a broadening of the curves, preceding the shift toward blue. The small aperture lowest-illumination curve seems to indicate a drop in red sensitiveness, or a reversal similar to the one occurring with the large field flicker curves with the writer. These secondary effects occur at the lowest illumination, where measurements are difficult, and thus cannot be established so well as the high illumination phenomena.

The flicker curves show similarly different effects due to changed illumination at different field-sizes, for while all show an increase of red sensitiveness for low illumination, the effect is most marked with the large field. There is no apparent reversal of the direction of the shift, although at the lowest illumination there is an increase of blue sensitiveness accompanying the increase of red, with the largest aperture. This is on a parallel with the drop in red sensitiveness at low illumination for large field in the writer's case.

One point is not well shown on these curves (which were selected to illustrate extreme cases), viz., that the shifts plotted occur chiefly at low illuminations. The high illumination curve represents very closely the position of the curves for a large range of illumination. Comparative fixity is found, in fact, from about 70 I.U. up.

The highest illumination (270 units) was the highest it was practicable to work at with the tungsten lamp. Some measurements made at about twice this illumination by viewing an acetylene flame directly showed that no appreciable change in the curves took place beyond this point.

(d) *Effect of changing field-size.*

Plates II. and II. *a* (Pl. IV.) present the luminosity curves at each illumination, as obtained with each size of field. The chief facts are, first, that at the high illuminations change of field-size has practically no effect on the flicker curves. Second, decreasing the size of field increases the red sensitiveness of the equality of brightness method (at practically all illuminations) but decreases the red sensitiveness by the flicker method. In other words, just as there is a reversed Purkinje effect, with the flicker method, so there is a reversed "yellow spot effect." In the case of this effect, as with the changes due to changed illumination, it was necessary for the second observer (Plate II. *a*) in order to experience these changes, to go to lower illuminations than did the first observer.

It is questionable whether the changes plotted in the high illumination curves for the equality of brightness methods are true "yellow spot effect" changes, or are due to the uncertainty of the method. Some check curves made two months after the first of Plate II., while exhibiting the same fixed character of the flicker curves, gave the equality of brightness curves as substantially the same for all field-sizes. There is, therefore, good ground for believing that at the highest illumination here used the effects of changing field-size are small or absent by both methods, although the inherent uncertainty of the equality of brightness method may indicate changes even at high illuminations. At lower illuminations the changes are of an entirely different order of magnitude from the uncertainties of judgment, and so can be described positively. It appears then that the Purkinje and yellow spot effects are two phases of the same low illumination phenomenon. In brief, at low illuminations for large fields there occurs the Purkinje effect by the equality of brightness method; the reversal effect by the flicker. At these illuminations a decrease in the size of the field decreases the effects.

(e) *Relative positions of flicker and equality of brightness curves.*

Plates III. and III. *a* (Pl. V.) give the relative positions of the two kinds of curves for three field-sizes and four illuminations. The most striking phenomenon is the large difference in the curves at low illuminations. At high illuminations the curves approach each other. In the case of the first observer, practical coincidence of the two curves is obtained for the middle-sized field at the highest illumination. In the case of the second observer (Plate III. *a*) this coincidence does not occur, the curves merely being much nearer together and (as test measurements at higher illuminations showed) they have reached a "steady state." With regard to the different relative behaviour of the two curves with the two observers, the remarks made on "reproducibility" and the comparative results of five observers given below, are *à propos*. With both the observers whose curves have so far been shown, there has occurred during the progress of the work a change of criterion with the equality of brightness measurements, and in the case of the five observers some show the equality of brightness maximum to one side, some to the other of the flicker. The only conclusions that can be drawn are that the two curves change from wide disagreement at low illuminations to a much closer agreement (in position of maximum) at high illuminations—the closeness of the agreement being difficult to determine because of the uncertainty in the real position of the equality of brightness curves. One point of significance is that although with both observers the two curves have reversed their relative positions on the red side, and in the comparative measurements by several observers the curves are differently placed on the red side, it has never happened that the flicker curve has shown more blue or green sensitiveness than the equality of brightness. In the writer's case, the two curves have been obtained exactly alike within the errors of measurement on two occasions, at two different high illuminations, with the middle-sized field, but this is as near as the blue part of the flicker curve has ever come to crossing over the other. Whether this would always be the case can only be determined by numerous measurements either extended over long intervals or by numerous observers.

It is possible that this phenomenon of lower green sensitiveness by the flicker method may be intimately connected with the fact that the comparison colour throughout was a yellowish white. Certain experiments not yet completed seem to indicate a dependence of the shape of the luminosity curve by equality of brightness on the colour of the comparison lamp. It has been found, for instance, that under certain conditions a spectrum yellow and a spectrum green may each appear by equality of brightness of the same intensity as a spectrum red, yet not appear equally bright when compared directly with each other. One might think, therefore, that a three-part field could be constructed in which two colours each appeared equally bright with the third, at their edges of contact with it, but not equally bright to each other at their common edge of contact. These phenomena of simultaneous contrast are yet to be investigated in connexion with the equality of brightness method, and the question must be settled as to whether any similar effects occur with the flicker photometer.

Perhaps the most interesting feature in these curves is their area. Casual inspection reveals the fact that the areas of the flicker and equality of brightness curves are frequently not the same. Now if the total light is the sum of the parts, this would mean the total light greater with one photometer than with the other. If the spectrum measured were that of a "4-watt" lamp exactly like that used as a standard of intensity, it would be possible to recombine the dispersed spectrum and make a homochromatic comparison of the two intensities of standard lamp and measured spectrum. This would of course be the same by both photometers. In other words, the physical summation must measure in this case the same, but by our curves the arithmetical summation is different by the two methods. One method or the other, perhaps both, must be deficient as judged by the ability to add or integrate the measured quantities. This question is being investigated further.

(f) *Comparative results obtained by different observers.*

When these measurements were finished it was thought well to obtain some measurements by other observers in order to serve as a check, and also to obtain data on differences to be expected between observers. Five observers in all were available, and these are indicated by initials. H.E.I. and M.L. are the two having previous experience with the apparatus; P.W.C., no previous experience with apparatus, some experience in photometry, considerable experience in observing with various optical instruments; F.E.C., long experience in photometry; C.F.L., considerable experience with optical instruments and some in photometry. These observers have all perfectly normal colour vision. Before these measurements each observer made a number of settings with a three-colour mixing instrument (Ives' colorimeter) to match white. From these settings it appears that the observers differed from the mean by no more than 5 per cent. in the proportions of red, green, and blue necessary to make the match. Differences of this amount are found by the writer to be the rule with all who fall in the class of "normal vision." Those occasional observers who would be classed as partly colour blind will differ by forty or fifty per cent. from the writer or from the mean of several "normal observers."

Two illuminations were determined on, a high one of 250 units, a low one of 10 units, and the middle size of field was used. This choice of illumination and field-size was made in view of the previous work. The high illumination is one beyond which no changes in the relative positions of the curves are to be expected, the low illumination one at which flicker measurements are still easy to make. The middle size of field was chosen partly because it is the easiest one at which to make observations, and partly because the nearest coincidence of the two kinds of curves had been obtained for this size.

The apparatus was left undisturbed between experiments. Except in the case of the observers H.E.I. and M.L., the divided head on the slit was read by an assistant. The tungsten lamp was a new one which was given a short seasoning, and at the conclusion of the measurements its energy distribution was immediately measured, as before described.

Plate IV. (Pl. V.) exhibits the effects of changing illumination for the two methods, for each observer, with the mean curve of all. The chief phenomena of the previous measurements are here borne out. With the flicker method, decreased illumination increases the red luminosity; with the equality of brightness method the same change increases the blue luminosity (observer M.L. at this illumination shows merely a broadening of the curve). In the case of the less experienced observers the equality of brightness measurements are more or less wild, the flicker measurements very good.

Plate V. exhibits the relative positions of the two kinds of curves for each observer, with the mean. At the low illumination the equality of brightness curves are all higher on the blue side than are the flicker. At the high illumination two observers place the flicker maximum on the red side, two on the blue side of the equality of brightness maximum; one observer places the maxima about together, although, as has before been noted, the curves do not actually coincide, being of different areas.

The mean high illumination curves obtained by the two methods show close agreement in the position of the maxima, and it is not unreasonable to expect that with more observers or more observations extended over such an interval that the observers would not remember their previous equality of brightness criteria, the two mean curves would show practical agreement as to shape and position of maxima. On the other hand, it is apparent that an exact coincidence of the curves may not confidently be expected, but rather two curves of the same shape of slightly different areas.

Figs. 7, 8, 9, and 10 (Pl. III.) give the luminosity curves of all observers plotted together. It is obvious from these that even with workers of normal colour vision considerable differences may be expected (of the order of five to fifteen per cent.) in their measurement of pure colours against an unsaturated colour such as the light of an incandescent lamp. The observers maintain substantially the same relative positions with respect to each other, so that the different positions of their luminosity curves probably represent real differences in colour vision. The flicker curves lie closer together than the equality of brightness, so that in general the observers will differ less on their flicker measurements than on the others.

It is clear, however, from these curves that whichever method of measurement is employed, we cannot expect to obtain results in exact agreement from observers taken at random, where lights of different colour are to be compared.

The disagreement between observers will be less in comparing the unsaturated colours of any acceptable illuminants than in measuring bright spectral colours, but it is nevertheless evident that in establishing standards of different coloured lights, either the mean of a number of observers of "normal colour vision" must be taken, or the luminosity curve of each observer must be known in order that corrections may be made to his readings—when it is learned how to apply such corrections.

Fig. 11 shows the distribution of energy in the source used in the comparative measurements* (tungsten lamp through prism, &c.) and the mean flicker curve of five observers at 250 I.U. reduced to an equal energy distribution. Since the flicker curve closely agrees in shape with the equality of brightness curve, but is more definitely determined, it may be taken as the mean luminosity curve of these observers, and probably lies closer to being the luminosity curve of the normal or average eye than any heretofore obtained. Its maximum lies at $\cdot 545\,\mu$. This is in very good agreement with the high illumination curve obtained by Kœnig for his own eye†.

Summary of results.

The chief new experimental results of this investigation are:—

1. The flicker method is more sensitive than the equality of brightness method, where different coloured lights are compared.
2. The results by the flicker method are more reproducible than those by the equality of brightness.
3. Decrease of illumination shifts the maximum of luminosity toward the blue, by equality of brightness (Purkinje effect); toward the red by the flicker method.
4. Decrease of the size of the photometric fields at low illuminations shifts the maximum of luminosity toward the red for the equality of brightness method (yellow spot effect); toward the blue for the flicker method.
5. The relative positions of the two kinds of spectral luminosity curves are in general different.

* The distribution of energy in the source used for Plates I. and II. (Pl. IV.) for an older tungsten lamp is slightly different from the lamp used in the comparative measurements.

† The value of $\cdot 565\,\mu$ for the maximum as obtained by Nutting for Kœnig's observations left out of account the correction for the dispersion of the prism.

6. The curves are most different in position at low illuminations with large fields; nearest together at high illuminations and with small fields. They may under certain conditions coincide, and the mean curves of several observers show close agreement in position of maxima and shape of the two curves at high illuminations, although the areas are not the same.
7. The curves obtained by different observers show different positions for each curve, and different relative positions of the two for high illuminations. At low illuminations all observers agree in showing the Purkinje and reversed Purkinje effects above described.

Theoretical Considerations.

No satisfactory theory of the action of the flicker photometer can be said to exist. What does it actually measure? We may assume the existence of a "luminosity sense" distinct from the colour sense. We may then form a geometrical picture of the alternations of luminosity as occurring in one plane, those of hue in another. In the second plane the sensations fuse more quickly, so that the sensation of flicker due to difference of luminosity persists longer than that due to difference in hue. Certain experiments by Tufts on the fatigued eye give some warrant for the assumption of this luminosity sense. The occurrence of colourless after images from coloured stimuli may also support this conception. The luminosity sense may be identified with the black-white element of Hering's theory. But at present it is questionable whether this luminosity sense can be considered as much more than a name used to assist in picturing the action of the flicker photometer. If it should be established that there does exist a separate "luminosity" element, which could be separated from hue by some photometric method, it would still remain to be established that this corresponded to our ordinary idea of brightness. If, for instance, there exists a physiological process called into action both by coloured and uncoloured light, a measure of this would be a measure of a common property. This might be satisfactorily termed "brightness" by a physiologist, but might not satisfy the photometrist who is interested not so much in the ultimate analysis as in things as they appear.

Dow has given reasons for believing the action of the flicker photometer to be chiefly ascribable to the retinal cones. The rods are known to be more sluggish, to possess greater inertia, so that they are less sensitive to flicker. The fact that the flicker photometer cannot be used at low illuminations, and that, as Dow found, the yellow spot and Purkinje effects are less by it, would support such a belief. The existence of a reversed Purkinje effect and a reversed yellow-spot effect, as found in the present investigation, appear to render Dow's theory inadequate. While a shift from cone to rod action explains very well the known facts of the Purkinje and yellow-spot effects and fits with the distribution of rods and cones in the retina, the flicker phenomena would appear to call for a more complicated mechanism of seeing.

Up to the present, the various facts upon which a more complete hypothesis may be formed are :—1st, The large increase in blue sensitiveness at low illuminations for large fields, and the smaller increase for small fields, by the equality of brightness method. 2nd, The large increase of red sensitiveness at low illuminations for large fields, and the smaller increase for small fields, by the flicker method. 3rd, The greater sensitiveness of the flicker method at high than at low illuminations, and for central as compared with peripheral vision. 4th, The complete failure of the flicker method at very low illuminations. 5th, The higher speed necessary for a large field centrally viewed as compared with a small field, and as compared with a small peripheral field.

Certain of these facts seem to support Dow's contention that the cones are probably the chief seat of action with the flicker photometer. The failure of the flicker method at low illuminations, where the rods are coming into play, speaks particularly for this. But it is necessary to ascribe some complexity of structure and behaviour to the cones if they are to suffice. As has been established by study of the retina, the cones are scarcer towards the periphery, but at the same time change in character, becoming larger. If we ascribe to these peripheral cones the property of exhibiting the typical cone characteristics (red sensitiveness and sensibility to flicker) more decidedly as the illumination decreases, we can account for some of the phenomena described above. In equality of brightness measurements such greater red sensitiveness in the cones would tend to compensate for the increasing activity of the rods and to preserve conditions of vision constant. If the flicker method uses only the cones, decreased illumination would bring out this compensating action at illuminations where the equality of brightness method would show comparatively little Purkinje effect, both retinal elements still being in action. As a matter of fact the curves of Plate IV. (Pl. V.) for five observers at an illumination of 10 units do exhibit, on the whole, a more marked shift toward red by the flicker method than is the shift toward blue by the other method. To explain the low sensibility and low critical speed for peripheral vision—an apparent reversal of the effect caused by increasing the size of the field—it is only necessary to assume the cones to be present in too small number, as the distance from the centre of the field becomes increasingly large, to play an important part. We then obtain rod flicker, which is produced by low speeds of alternation.

Along some such lines as this it is possible that the phenomena of the flicker photometer may ultimately receive explanation. The present work has not been carried sufficiently far into extreme conditions of illumination and localization upon the retina to establish that the action of the flicker photometer is to be ascribed chiefly to the cones, although this may appear probable. Even were this established there would still remain the larger part of the problem. For how do the cones fuse the colour impressions previous to fusing the "luminosity" impressions? Were the facts exactly opposite, if intensity flicker disappeared first with increased speed and colour flicker persisted, we could ascribe the luminosity phenomena to the rods or organs of colourless vision which possess greater inertia, and the colour phenomena to the cones, in accordance with the commonly known properties of each. Unfortunately, we must depend upon the cones for both the colour and flicker photometer phenomena; hence the need for ascribing to them some complexity of function as suggested above, and the difficulty of comprehending how the separation of luminosity and hue takes place.

Morris-Airey has suggested that the differences between the flicker and equality of brightness photometers may be ascribed to the different rates of the rise of sensation with different colours, a phenomenon which could affect the flicker but not the other instrument. For instance, in the flicker photometer, when adjusted to show no flicker, if either light source is obscured, the violently flickering illumination from the other source gives a decidedly greater impression of brightness than if the speed is increased until flicker disappears, or if one-half the steady radiation from that source is viewed. This apparently increased brightness has been found different for different colours. If this phenomenon is taking place even in the absence of all flicker, when the flicker photometer is at its mid-position, then the different amount of the effect for different colours might cause a difference in the readings of the photometers.

As yet this theory has not apparently received any thorough experimental test. It is not established that at the same illumination the rates of rise of sensation are actually different for different colours, and phenomena of this sort are extremely apt to be influenced by the order of the illumination. Nor is it established whether these phenomena are functions of speed rather than of the presence of noticeable flicker. In other words, whether the disappearance of flicker by alternation with another colour does not destroy these effects as completely as does the disappearance due to higher speed with the one colour. There is much yet to be investigated here, and in view of the differences between observers in their results as to the differences between the two methods, it is clear that these phenomena of rise of sensation must be studied with the same eye and apparatus by which are made the flicker measurements in which they may play a part.

And so it appears that there is no theory of the flicker photometer sufficiently supported by known phenomena to be at present satisfactory. The data collected in this investigation are probably insufficient to throw much light on the actual processes at work in the retina, and the investigation up to the present has had for chief object, apart from the practical photometric information so urgently needed, the location of landmarks from which to start the more searching investigations into the nature of the flicker phenomena and the brightness sensation.

Investigations which are now being carried on, or are planned with the same apparatus, are expected to furnish data which will assist in the formation of satisfactory theories. Prominent among these investigations may be mentioned: Luminosity curves as given by the method of critical frequency; the comparison of the physical and arithmetical summation of measured illuminations; the effect of changing the hue of the comparison source; the phenomena of rise of sensation as they occur in the flicker photometer as used; the connexion between hue difference, speed, and sensibility in the flicker photometer.

Bearing upon Practical Photometry.

The primary object of this investigation is to decide upon the photometric method and conditions to hold in determining the candle-power values of lights of different colours. Until the completion of the work laid down the full specification of these conditions cannot be made. There is as yet no answer to the question—What is the candle-power of a light in terms of a standard of different colour? Certain conclusions may, however, be drawn at the present stage of the work which will hold no matter what the final answer to the main question. One of these is that the sensibility and reproducibility of the flicker method are sufficiently greater than those of the other method to recommend its use in all cases where colour differences exist and where the question of absolute intensity values is not of the first importance. In the determination of distribution curves of a source, for example, a flicker photometer would be very convenient. A second conclusion is to be drawn from the comparative data by several observers of normal vision. From their differences of reading it is apparent that differences of colour vision will always be a serious obstacle to uniform results in heterochromatic photometry. The only practicable escape from this difficulty is to eliminate the need of making such comparisons in ordinary photometric practice; in other words, aim to make all practical photometry the photometry of lights of the same colour.

Under these conditions it is a matter of indifference what photometer is used, or whether the observer has normal colour vision. The problem of heterochromatic photometry hence becomes one for the standardizing laboratory, where secondary coloured standards or coloured glasses will be prepared for as many practical cases as possible. The search for a photometric method for coloured lights will in this work be treated from that standpoint.

Of considerable practical interest is the fact that the flicker method most nearly agrees with the method of equality of brightness at high illuminations. Until the problem is further toward solution it would appear well, whatever method is chosen, to make all heterochromatic comparisons at high illuminations. An idea of the order of magnitude of the illumination here meant may be obtained by noting that all the marked shifts (Purkinje phenomena, &c.) occur below 70 I.U. This corresponds to about 1/10 that number of metre candles as viewed by the normal sized pupil, or approximately, seven metre candles.

Acknowledgement.

The writer takes pleasure in acknowledging his indebtedness to Dr. P. W. Cobb, Dr. C. F. Lorenz, and Mr. F. E. Cady for their kind assistance in making certain of the measurements here recorded and used; and especially is he indebted to Mr. Matt Luckiesh, not only for making many of the readings but for the preparation of the numerous drawings and the calculation and reduction of results.

Physical Laboratory,
National Electric Lamp Association,
Cleveland, Ohio, December 1910.

Table I.—Data for Plates i., ii., and iii. (Pls. IV. & V.)

The flicker luminosity curve for the $4\frac{1}{2}°$ field is reduced to the value unity at ·574 μ at each illumination, and the other data are given in terms of this. The data are corrected for dispersion, but not for the energy distribution of the source.

λ.	Field 8°·6 × 5°·16		Field 4°·58.		Field 1°·86.	
	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.
			Illumination 270 Units.			
·657 μ	...	10·5	...	...	...	...
·565	14·3	...	14·2	11·5	13·3	13·3
·643	26·9	21·5	27·0	25·3	25·0	26·8
·632	42·9	38·8	42·3	41·7	42·0	43·7
·622	58·8	56·2	58·4	57·5	55·3	61·3
·6125	74·0	71·6	73·9	74·7	70·6	77·9
·593	98·4	98·8	96·0	97·8	98·4	97·1
·574	100·5	103·1	100·0	98·6	100·0	100·0
·555	89·0	96·8	92·1	92·6	91·1	93·3
·545	81·9	88·8	81·4	81·7	81·0	81·7
·535	66·8	74·3	66·0	67·2	66·5	69·3
·526	50·9	59·0	51·0	51·0	49·3	49·8
·517	34·3	36·1	33·0	32·7	32·0	32·4
			Illumination 68 Units.			
·655 μ	...	...	14·0	17·4	15·2	...
·654	15·6	18·8	...	...	...	19·3
·642	30·4	30·7	29·0	31·1	30·4	34·4
·632	44·6	46·0	44·1	49·9	45·5	50·5
·622	59·6	60·3	59·1	65·4	59·6	65·3
·612	74·7	76·5	73·5	81·3	73·4	78·6
·593	98·3	95·2	95·0	100·5	93·9	99·6
·574	99·0	96·8	100·0	102·2	101·6	100·5
·555	87·8	93·2	89·8	96·3	88·3	91·2
·545	77·0	81·9	80·8	87·5	79·4	81·7
·535	64·5	70·6	67·9	74·7	66·6	69·2
·526	50·0	57·4	52·2	57·5	50·9	52·7
·517	33·3	37·7	35·5	37·4	31·5	32·0
			Illumination 8·9 Units.			
·6525 μ	21·6	20·4	20·2	19·2	23·0	25·3
·642	37·6	34·4	35·4	37·4	37·9	43·8
·632	51·1	47·1	51·9	51·1	53·4	59·1
·622	71·1	68·4	65·8	69·0	68·6	80·1
·612	83·0	79·8	79·7	79·0	82·4	95·3
·593	101·6	94·5	101·6	101·5	102·8	116·7
·574	94·9	102·2	100·0	102·3	106·4	115·2
·555	84·2	103 9	87·9	108·8	93·4	101·1
·545	71·6	97·4	75·6	98·0	79·3	91·2
·535	58·1	90·3	61·0	88·2	67·0	78·9
·526	44·8	75·6	48·4	70·5	50·5	59·9
·517	27·7	54·3	30·9	50·1	31·6	40·1
·508	...	31·1	20·9	28·4	19·3	22·5

TABLE I. (*continued*).

λ.	Field 8°·6 × 5°·16. Flicker.	Field 8°·6 × 5°·16. Eq. of B.	Field 4°·58. Flicker.	Field 4°·58. Eq. of B.	Field 1°·86. Flicker.	Field 1°·86. Eq. of B.
Illumination 2·85 Units.						
·653 μ	20·2	14·2	20·9	17·5	22·6	23·2
·643	31·1	25·1	33·3	29·2	35·8	35·6
·632	48·5	43·7	48·7	44·9	49·6	48·1
·622	66·0	53·2	67·8	54·8	64·5	65·2
·612	75·4	65·2	80·3	68·9	78·4	77·5
·594	95·1	78·1	95·3	81·5	93·4	95·0
·574	95·9	85·9	100·0	90·6	95·9	93·7
·555	76·6	89·7	78·2	90·6	78·8	82·0
·545	66·8	86·6	66·1	83·2	71·3	76·6
·535	54·3	79·5	54·0	75·8	57·5	67·5
·526	39·2	62·0	37·5	61·6	45·9	54·3
·517	27·9	55·6	25·6	44·7	28·0	35·0
·5075	...	31·9	...	26·0	...	...

TABLE I*a*. Data for Plates I.*a*, II.*a*, and III.*a*.

The flicker luminosity curve for the $4\frac{1}{2}°$ field is reduced to the value unity at ·574μ at each illumination, and the other data are given in terms of this. The data are corrected for dispersion, but not for the energy distribution of the source.

λ.	Field 8°·6 × 5·16°. Flicker.	Field 8°·6 × 5·16°. Eq. of B.	Field 4°·58. Flicker.	Field 4°·58. Eq. of B.	Field 1°·86. Flicker.	Field 1°·86. Eq. of B.
Illumination 270 Units.						
·655 μ	14·0	12·0	13·4	12·9	13·1	12·7
·643	24·6	......	24·9	22·8	26·3	24·9
·632	38·5	38·8	41·3	38·8	41·8	40·1
·622	52·7	55·0	53·0	50·4	53·7	51·8
·612	68·9	64·8	68·2	62·7	67·5	65·7
·594	88·0	85·2	88·6	90·2	88·7	92·5
·574	100·4	99·2	100·0	100·8	98·5	105·4
·555	93·7	100·1	91·8	98·0	91·1	100·1
·545	83·0	87·0	82·7	91·4	81·8	89·1
·535	70·2	80·1	71·5	80·6	72·5	78·7
·526	52·9	62·9	55·4	65·8	53·5	61·4
·517	38·0	40·2	35·6	42·4	35·4	40·6

λ.	Field 8°·6 × 5°·16.		Field 4°·58.		Field 1°·86.	
	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.
Illumination 8·9 Units.						
·653 μ	20·5	12·6	18·9	15·1	17·7	16·3
·643	35·2	19·4	32·8	25·9	33·3	30·8
·632	52·6	30·0	48·5	42·4	44·6	45·8
·622	66·4	42·6	65·8	54·7	59·7	59·2
·612	78·0	60·9	81·4	66·3	71·4	72·0
·594	97·6	87·2	96·4	90·1	91·1	97·0
·574	101·1	92·8	100·0	100·0	98·3	98·9
·555	90·5	97·7	90·6	96·8	91·1	96·3
·545	78·7	90·0	80·4	87·1	53·1	83·0
·535	63·5	79·5	65·7	81·2	66·9	75·6
·526	50·5	68·4	53·0	65·0	53·1	59·8
·517	33·2	49·3	33·8	43·9	34·9	41·3
Illumination 2·85 Units.						
·653 μ	...	11·4	26·1	14·1	21·5	20·9
·652	28·8	...	...	...	...	...
·643	...	20·0	...	...	...	...
·642	39·5	...	38·9	27·6	34·1	32·4
·632	58·7	32·8	53·7	37·0	48·3	46·4
·622	69·5	42·8	69·6	52·7	63·3	62·0
·612	84·4	56·6	75·8	65·4	76·7	75·8
·594	98·7	82·0	94·7	82·0	97·3	96·7
·574	101·2	89·0	100·0	95·3	98·8	101·2
·555	80·3	92·8	87·7	99·1	89·1	95·7
·545	75·6	85·1	72·1	88·6	79·4	89·4
·535	57·5	81·0	63·2	78·5	66·1	75·5
·526	48·0	61·7	50·9	66·5	52·7	62·2
·517	35·9	53·8	34·8	48·8	35·0	40·0
·5075	...	40·6	...	32·5	...	...
Illumination 1·25 Units.						
·655	...	6·69				
·653	...	...	23·0	14·0	21·6	20·5
·652	26·0	...	...	...	...	..
·643	...	13·9	...	...	...	...
·642	...	...	33·0	24·6	34·4	31·5
·6415	37·2	...	...	...	...	...
·6325	...	21·6	...	...	...	...
·632	...	...	48·2	37·0	53·8	46·8
·6315	57·3	...	...	...	...	...
·622	...	31·1	61·3	48·4	62·8	60·5
·6215	69·4	...	...	...	...	...
·612	77·7	39·4	72·2	57·6	75·3	67·9
·594	88·0	61·5	96·0	74·5	99·9	90·1
·574	83·8	71·3	100·0	88·6	101·7	107·0
·555	71·8	77·4	83·1	81·2	88·3	97·8
·545	60·2	78·5	66·3	85·0	77·6	88·0
·535	53·7	73·8	56·1	83·4	66·2	72·5
·526	44·1	70·9	47·1	66·8	48·8	61·3
·517	37·3	54·2	39·4	51·6	32·4	46·6
·508	...	...	...	42·5	...	29·1
·4975	...	...	...	25·5	...	...

Table II.—Data for Plates iv. and v. (Pl. V.) and figs. 7, 8, 9, and 10.

Given corrected for dispersion.

250 I.U.

λ.	H.E.I.		M.L.		P.W.C.		C.F.L.		F.E.C.		Mean.	
	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.
·653 μ	16·3	18·5	16·3	12·6	25·6	31·2	22·7	46·3	19·4	38·8	20·0	27·3
·643	29·3	30·4	29·4	18·8	39·2	42·2	35·0	51·2	35·5	46·5	33·7	35·1
·632	42·4	46·4	39·1	32·6	52·9	63·8	46·6	85·5	45·8	52·6	45·3	53·5
·622	53·7	57·9	50·5	44·9	63·8	66·8	61·3	106·6	59·7	71·0	57·76	66·5
·612	66·2	73·5	62·9	58·1	74·8	80·0	70·1	75·8	70·2	70·0	68·75	68·9
·594	87·5	88·4	82·4	82·8	85·0	85·4	84·3	78·4	83·4	78·8	84·4	81·0
·574	91·0	90·5	90·9	93·1	84·7	86·9	85·5	67·1	84·1	89·4	87·21	85·3
·555	84·2	84·6	82·4	89·6	71·2	80·8	73·5	77·2	74·6	68·5	77·2	84·6
·545	73·5	75·9	74·4	81·2	62·6	72·6	65·4	63·5	64·8	59·9	68·0	72·6
·535	62·9	64·1	65·6	68·3	52·2	59·7	55·5	64·3	55·0	48·1	58·2	63·2
·526	50·8	50·8	53·5	58·6	38·6	49·1	41·7	43·0	41·4	31·0	45·2	48·4
·517	33·0	32·8	35·7	40·0	26·1	30·3	28·4	42·8	27·4	21·1	30·1	35·0

TABLE II*a*. Data for Plates IV. and V. (Pl. V.) and figs. 7, 8, 9, and 10.

Given corrected for dispersion.

10 I.U.

λ.	H.E.I.		M.L.		P.W.C.		C.F.L.		F.E.C.		MEAN.	
	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq. of B.	Flicker.	Eq, of B.	Flicker.	Eq. of B.
·653μ	19·5	16·6	18·7	11·5	29·1	29·5	26·6	39·0	29·2	27·2	24·6	24·9
·643	33·8	29·5	30·4	25·2	42·2	42·0	40·4	65·7	40·1	39·4	37·4	40·4
·632	47·9	43·2	45·9	39·3	57·7	60·2	56·6	82·0	56·5	58·3	52·9	56·4
·622	64·6	57·4	59·3	50·7	71·7	70·1	68·9	102·8	68·5	60·0	66·6	68·2
·612	75·6	69·2	72·2	63·3	84·8	83·2	80·0	99·3	81·6	76·2	78·8	78·2
·593	90·1	87·8	92·8	87·1	92·0	92·0	92·8	114·0	93·8	92·8	92·3	94·7
·574	92·7	90·6	94·3	93·0	86·3	92·6	88·6	96·2	86·9	93·5	89·7	93·2
·555	79·9	85·2	83·0	90·4	71·4	81·9	72·7	135·2	72·1	87·6	75·8	96·1
·545	67·9	77·8	74·0	83·4	62·6	78·0	60·9	125·0	63·5	81·2	65·8	89·1
·535	56·4	68·7	61·5	73·7	50·0	71·5	49·2	65·5	52·8	71·0	54·0	70·1
·526	43·8	54·5	48·5	60·0	35·1	56·8	36·7	108·8	40·5	54·6	40·9	66·9
·517	27·3	37·1	31·4	43·5	24·4	38·7	26·6	69·5	26·0	44·4	27·1	48·6
·508	19·0	23·0	21·0	26·9	......	24·3	......	28·2	......	......	20·0	25·6

TABLE III.

Mean Flicker Luminosity Curve for Five Observers. Reduced to Equal Energy Spectrum.

λ.	
653	10·6
643	19·5
632	28·8
622	40·2
612	52·8
593	78·7
574	100·0
555	111·0
545	112·0
535	111·0
526	99·7
517	76·7

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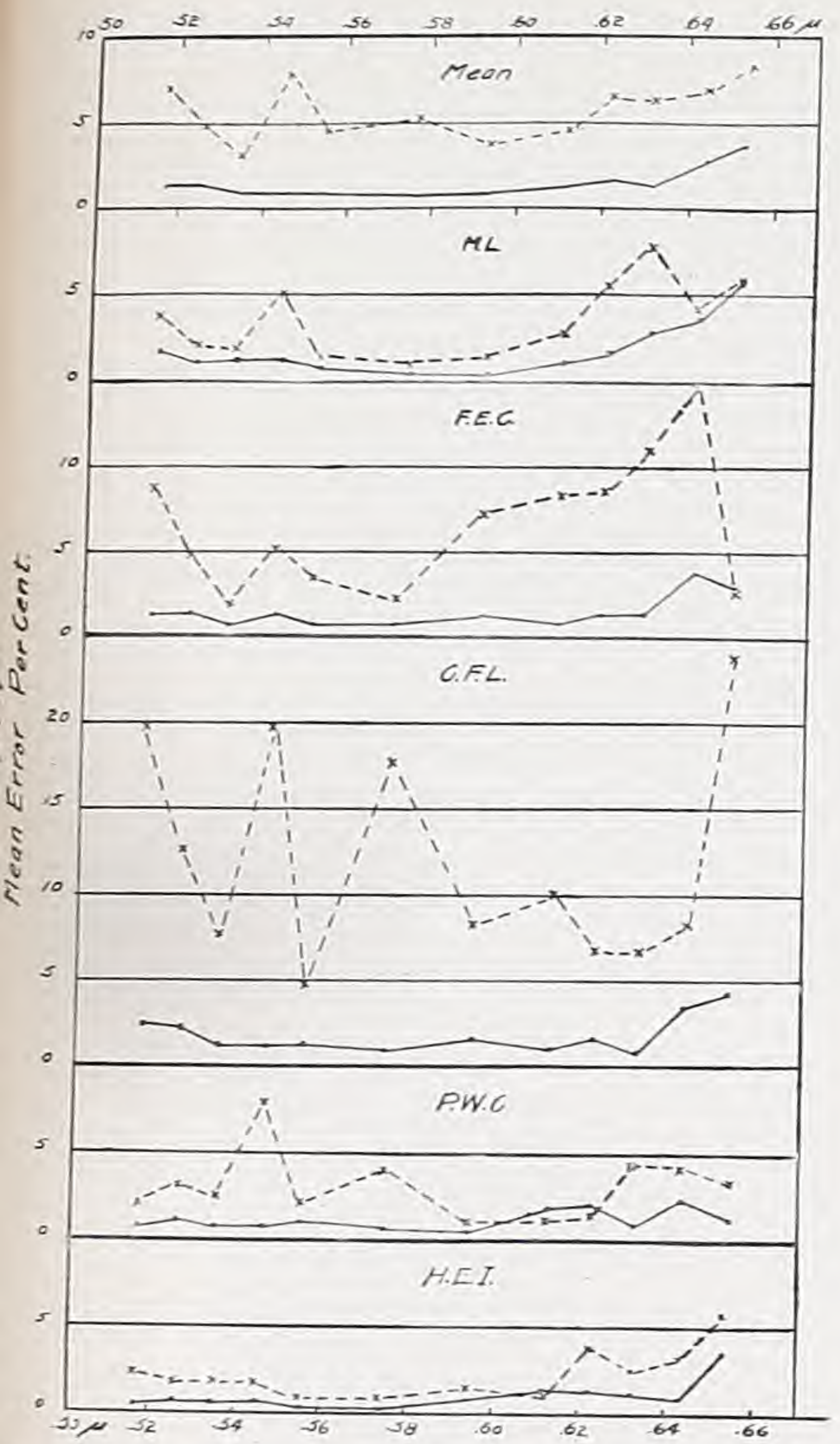


Fig. 2.—Sensibility of Flicker and Equality of Brightness Methods. (High Illumination.)

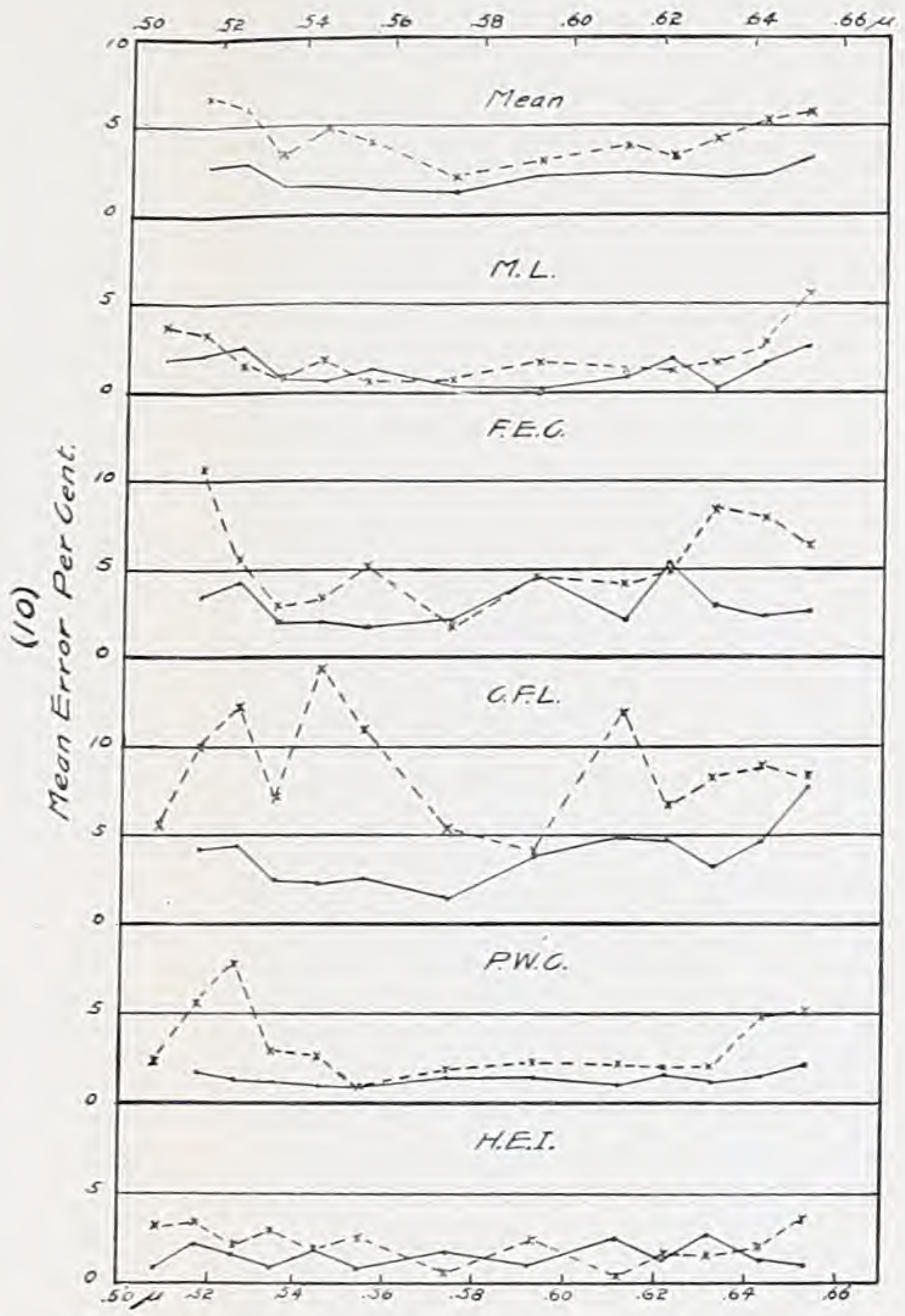


Fig. 3.—Sensibility of Flicker and Equality of Brightness Methods. (Low Illumination.)

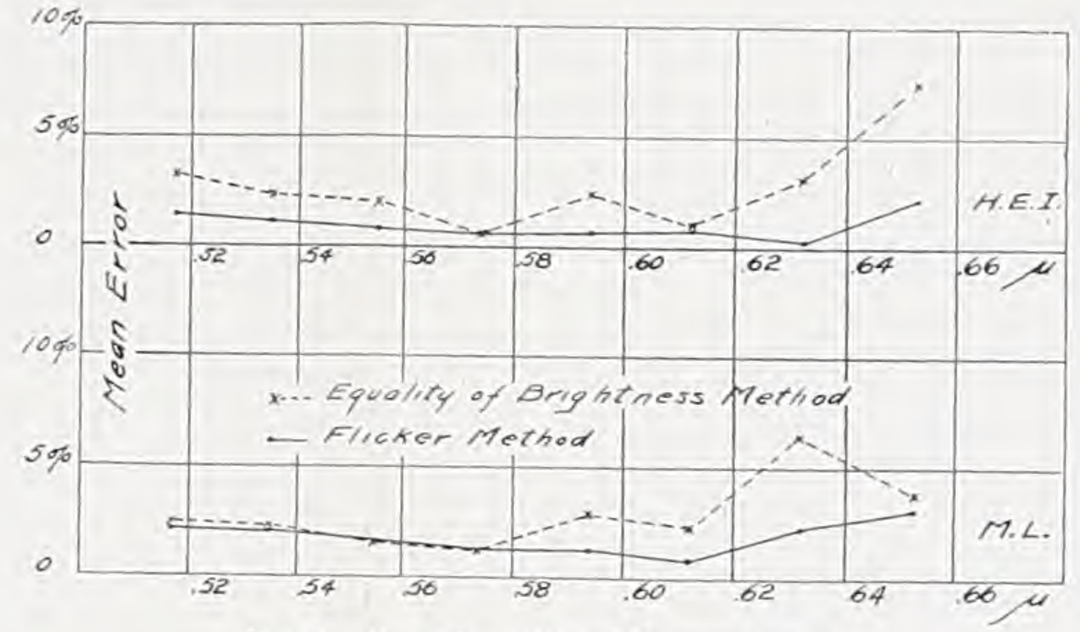


Fig. 6.—Reproducibility of Measurements. (Mean of 5 observations taken over a period of Two Months.)

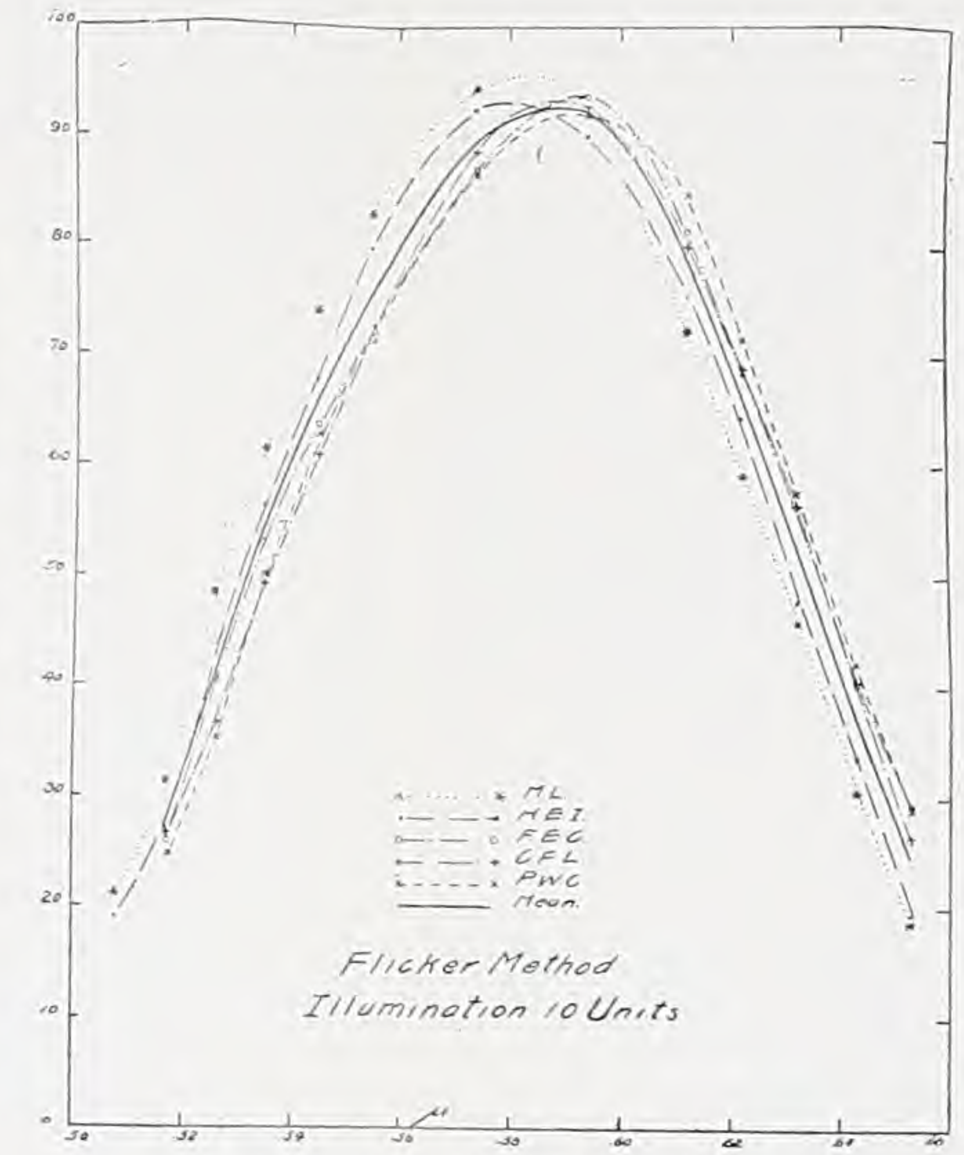


Fig. 7.—Luminosity Curves of Five Observers.

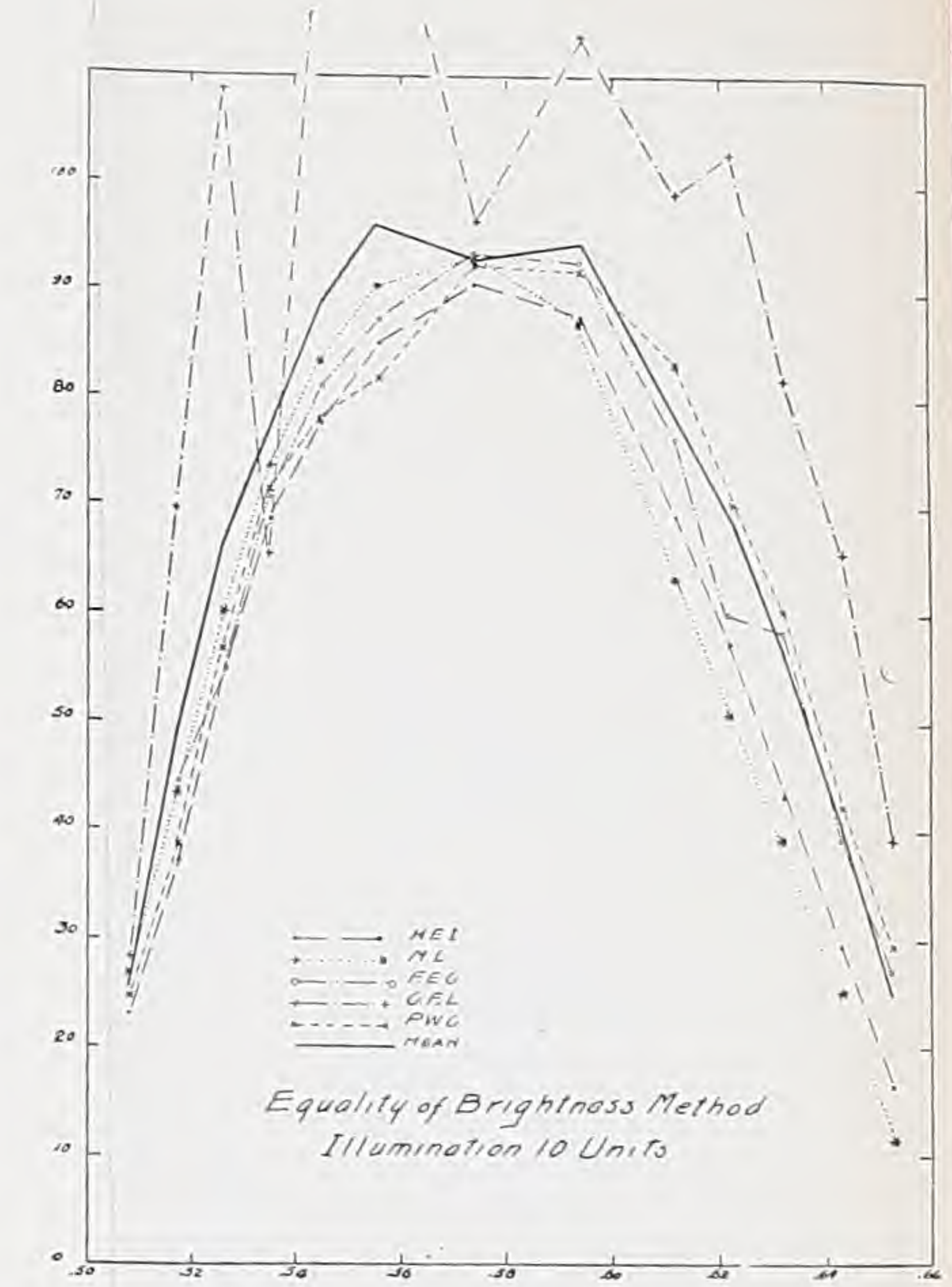


Fig. 10.—Luminosity Curves of Five Observers.

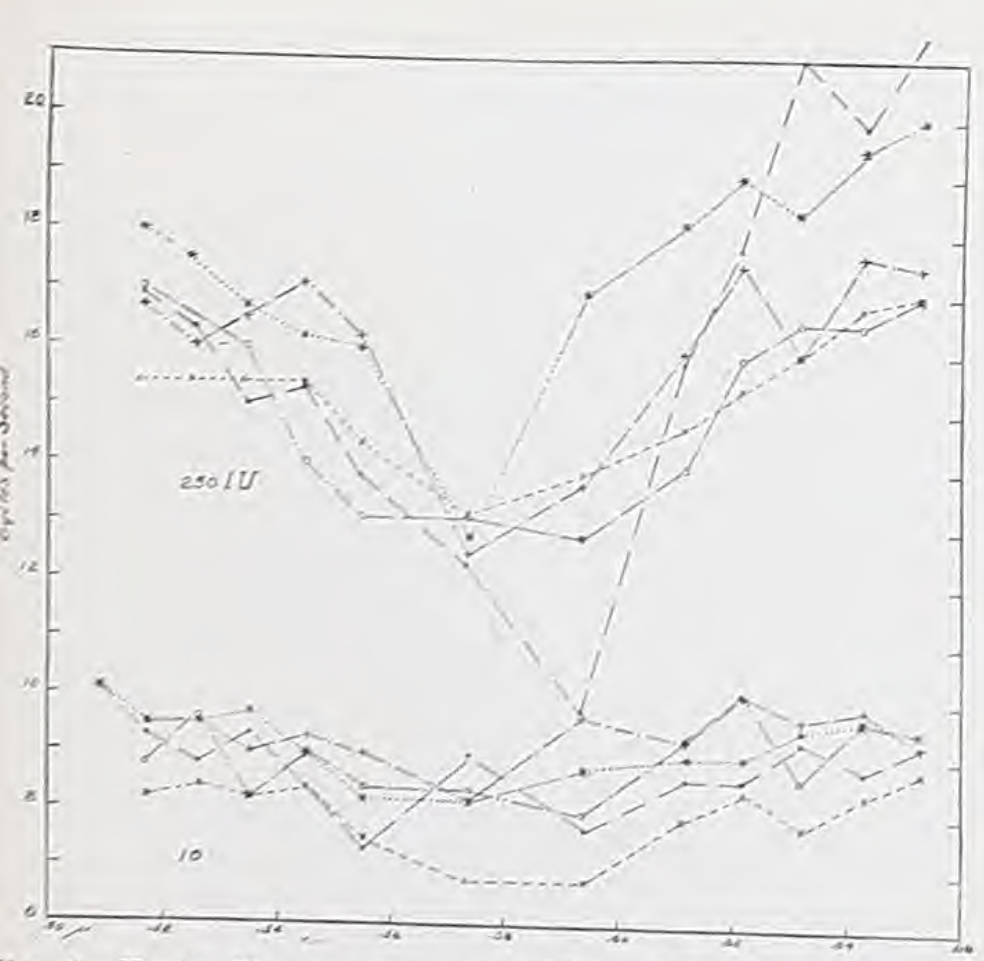


Fig. 4.—Flicker Photometer Speeds. (High and Low Illuminations.)

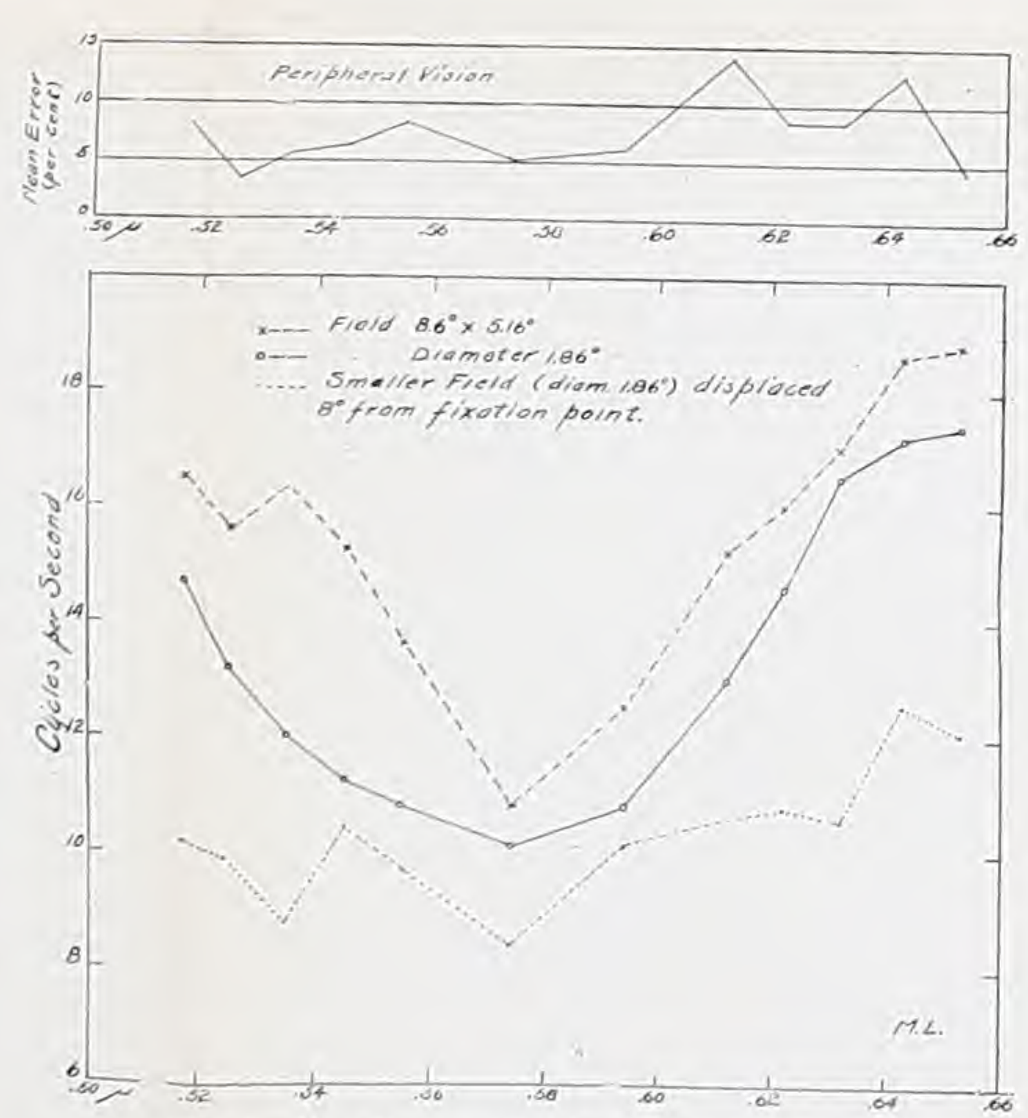


Fig. 5.—Miscellaneous Speed and Sensibility data.

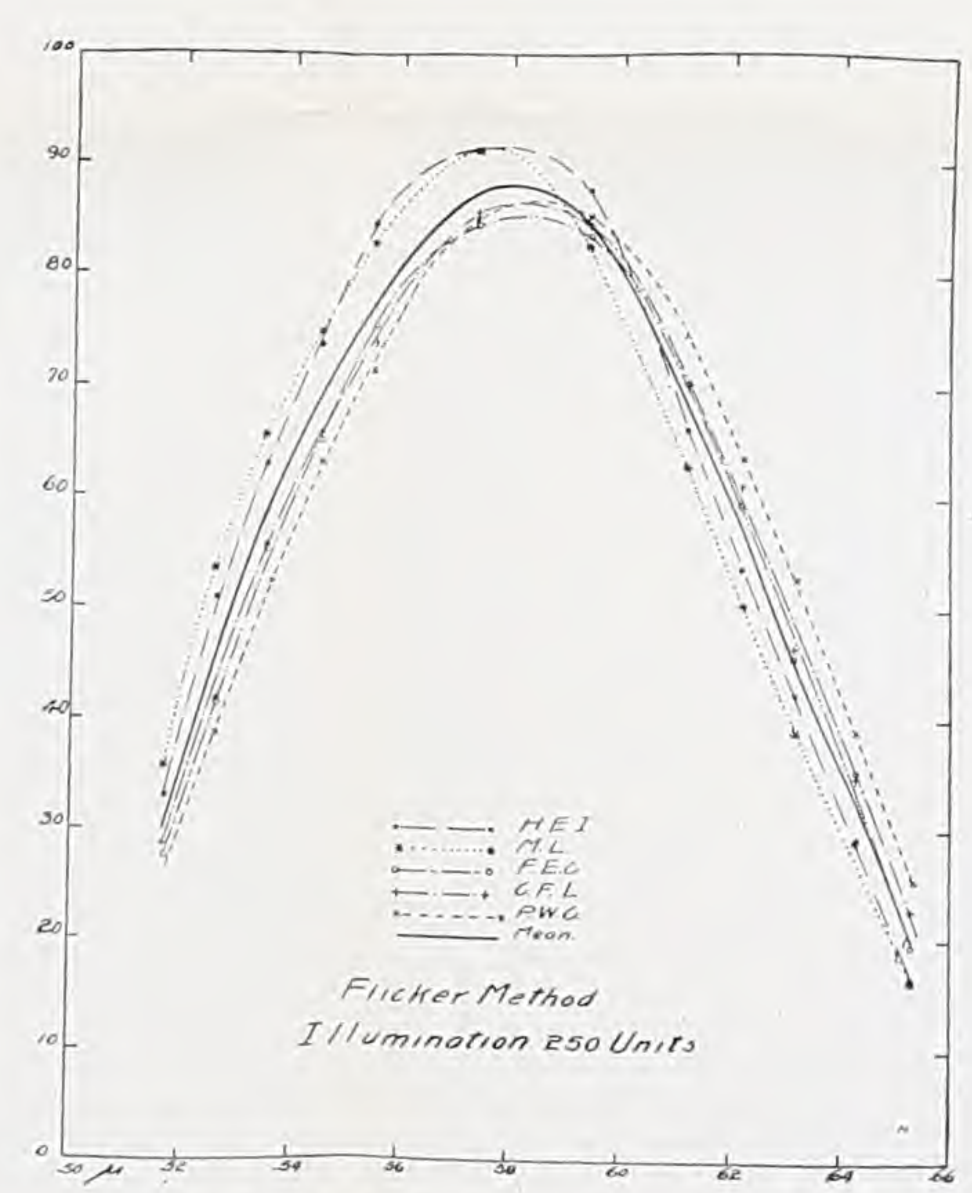


Fig. 8.—Luminosity Curves of Five Observers.

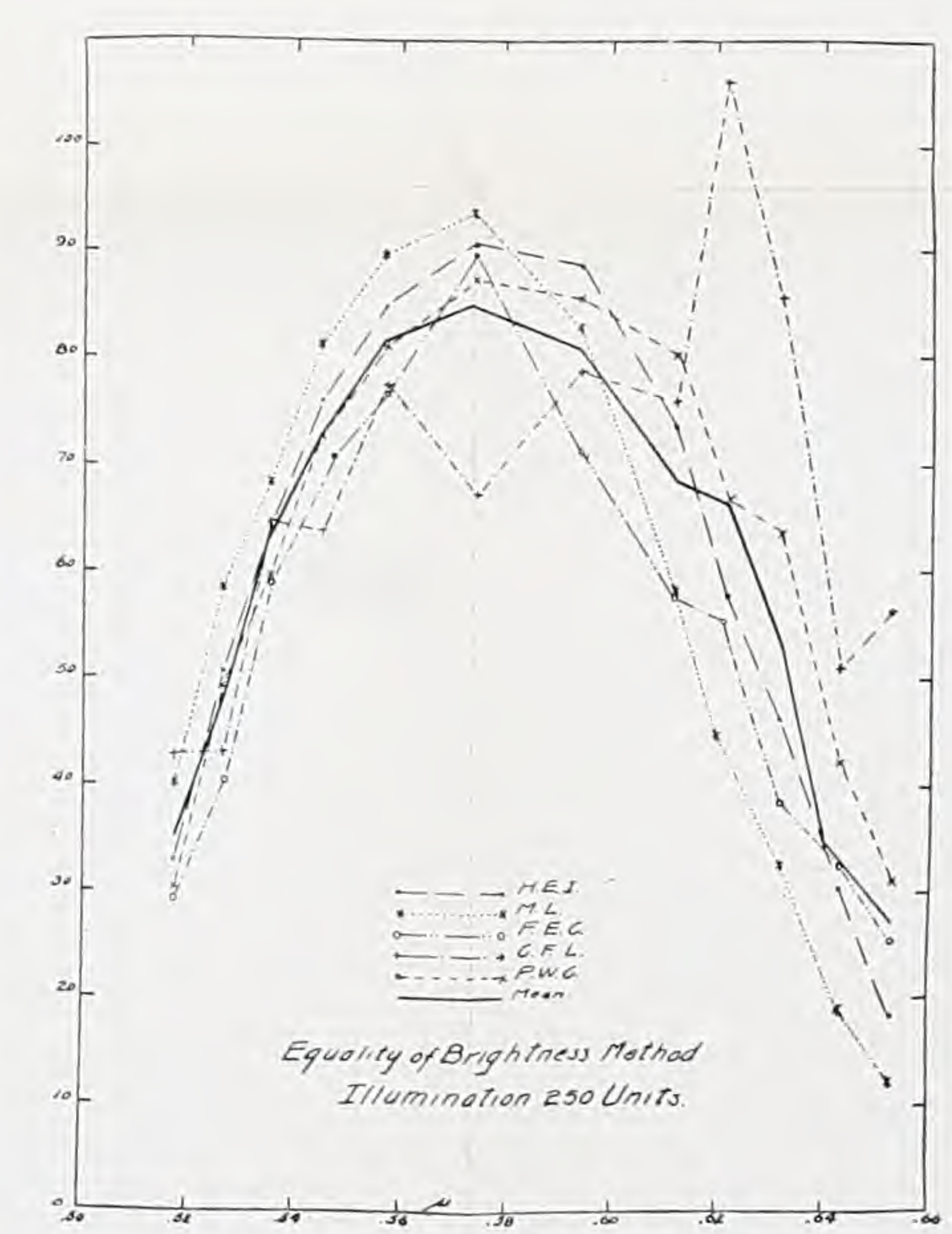


Fig. 9.—Luminosity Curves of Five Observers.

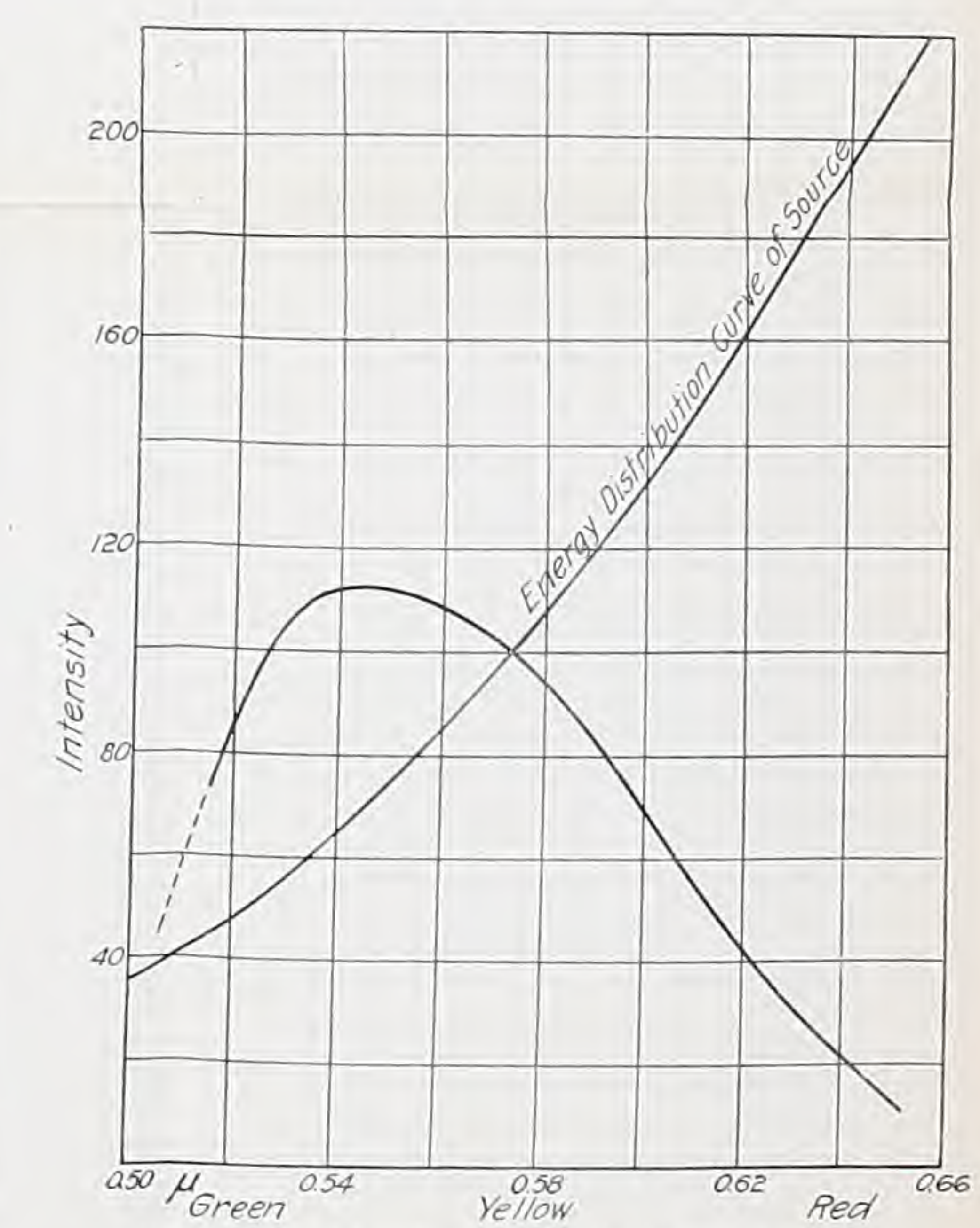
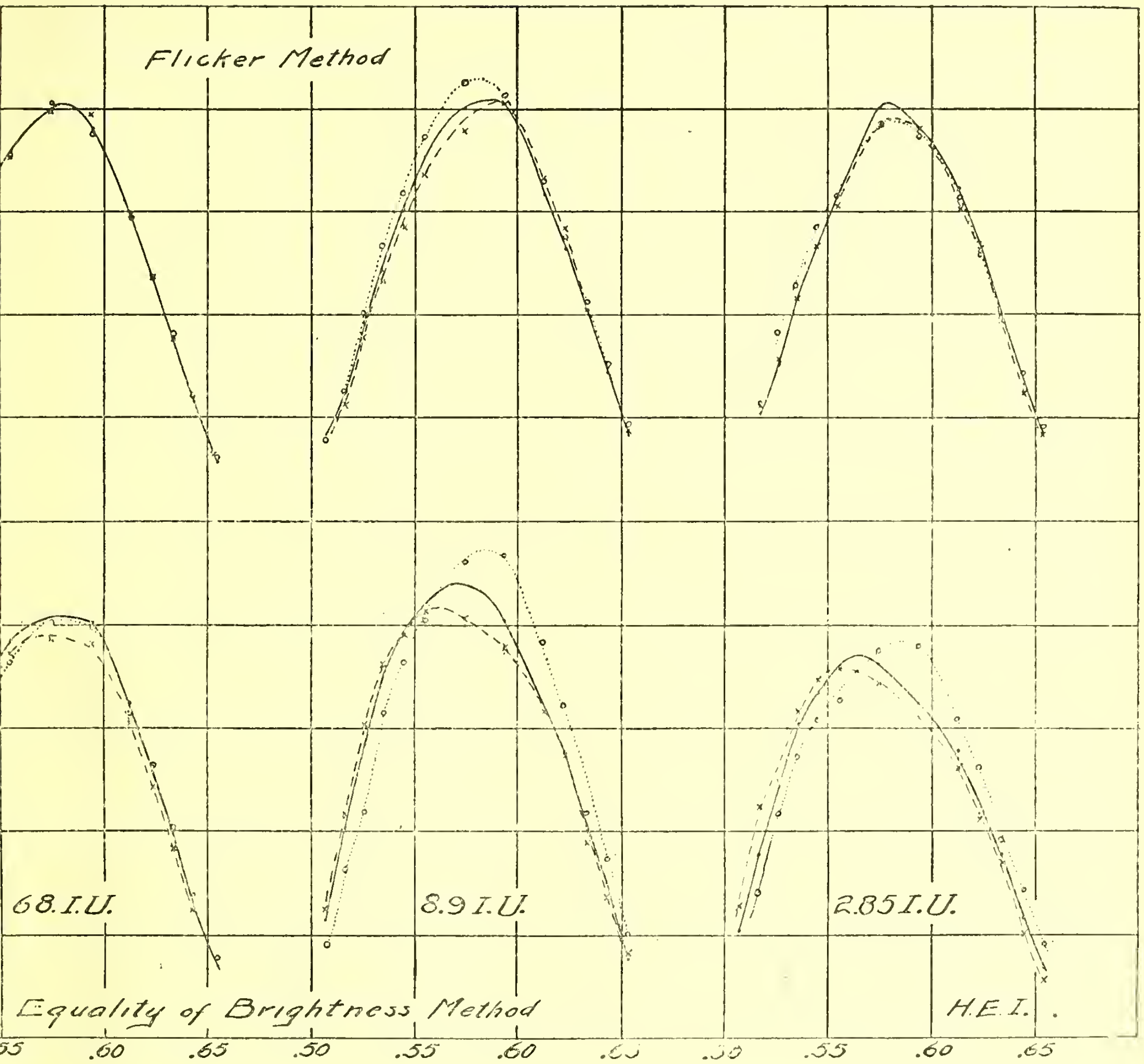


Fig. 11.—Mean Luminosity Curve for Normal Spectrum.



LATE II.—Effect of Changing Size of Field.

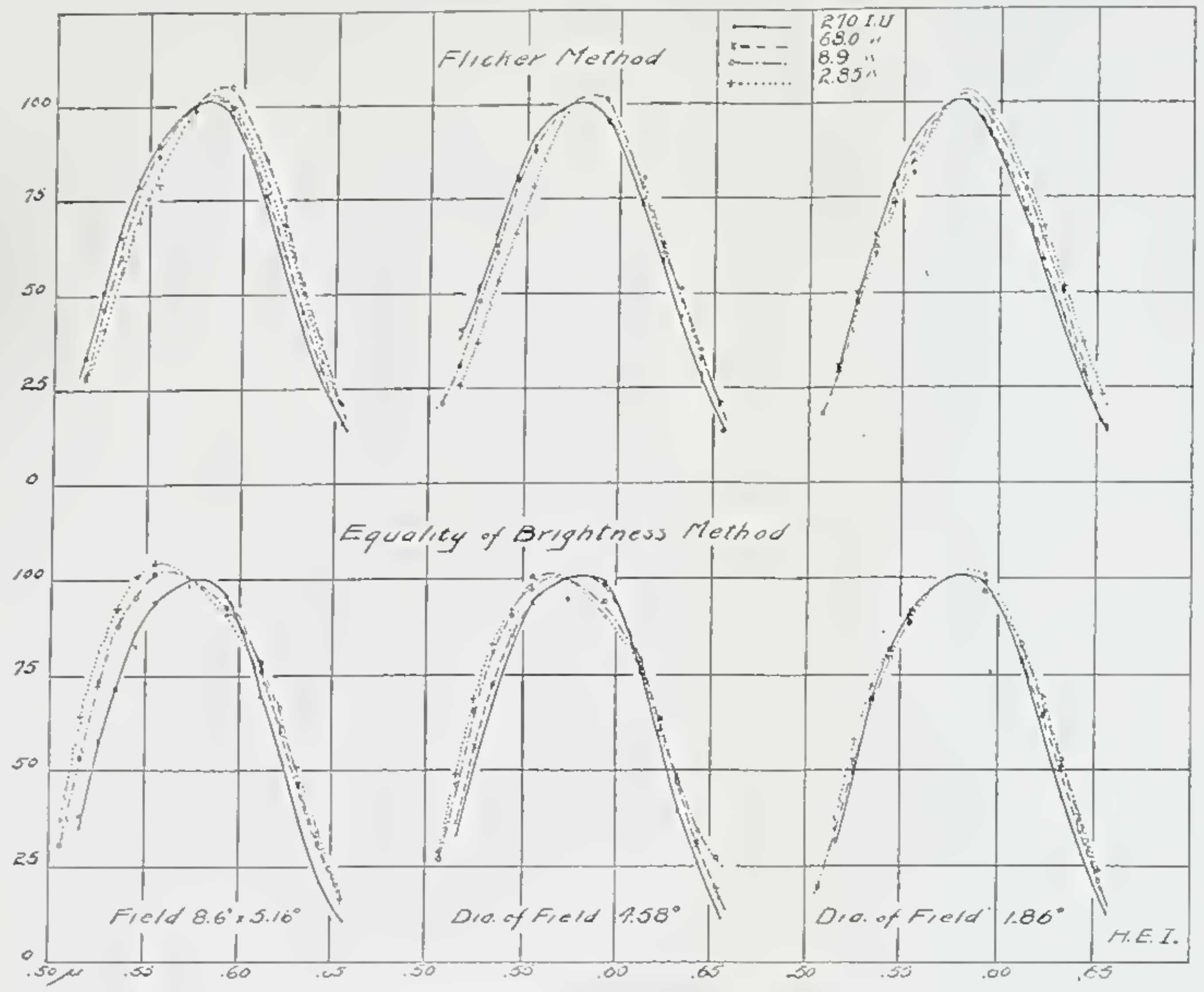


PLATE I.—Effect of Changing the Illumination.

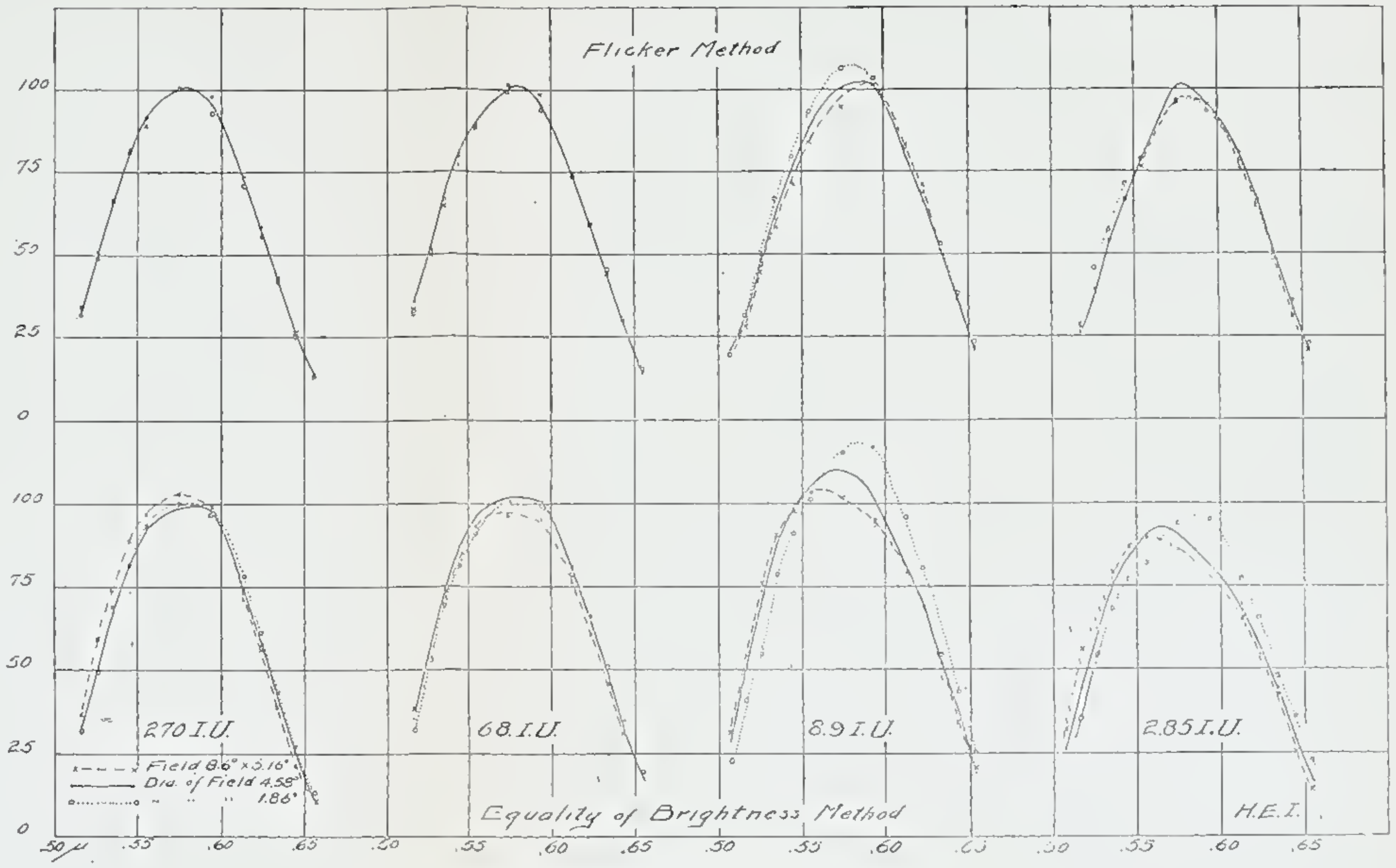


PLATE II.—Effect of Changing Size of Field.

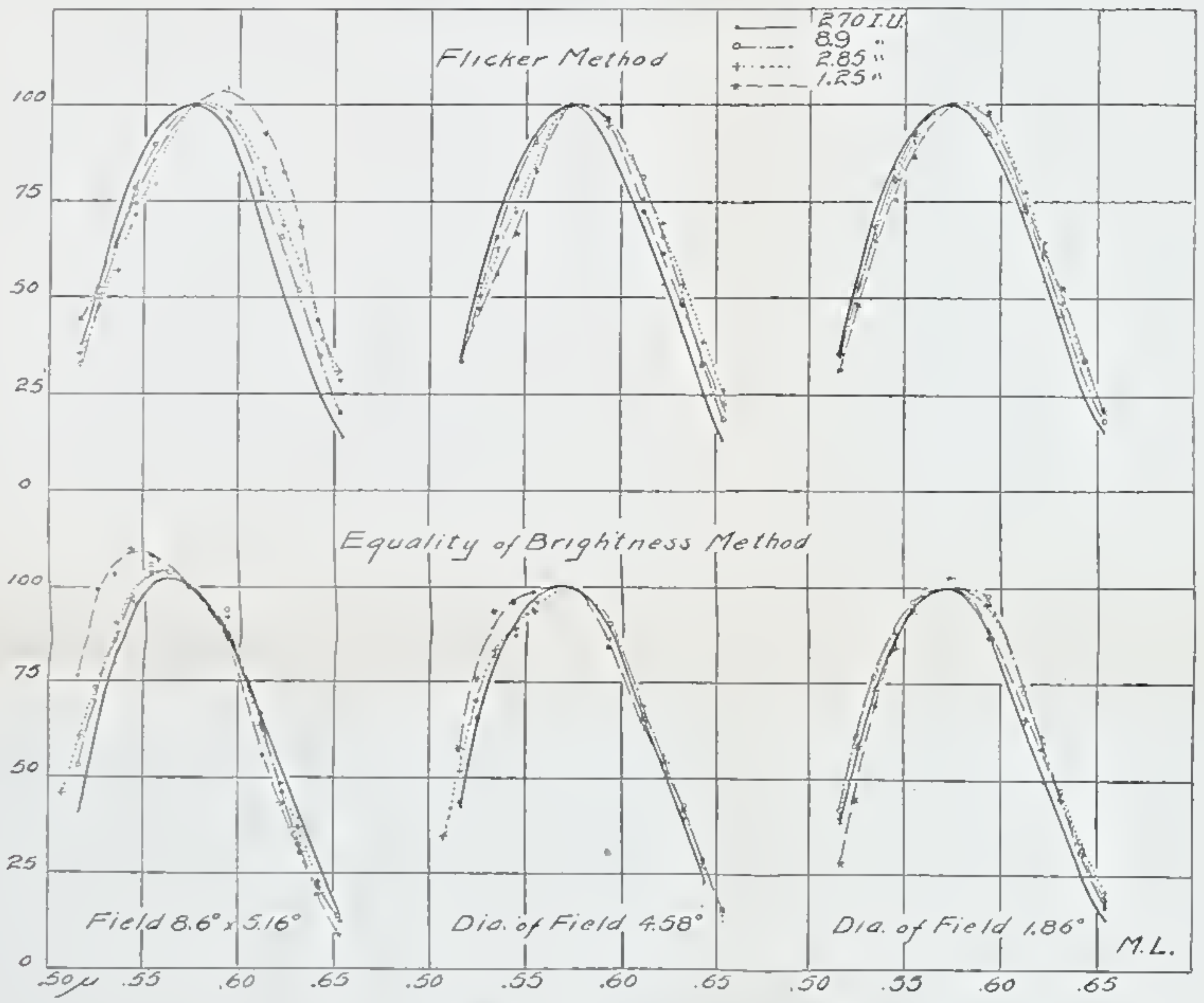


PLATE I *a*.—Effect of Changing the Illumination.

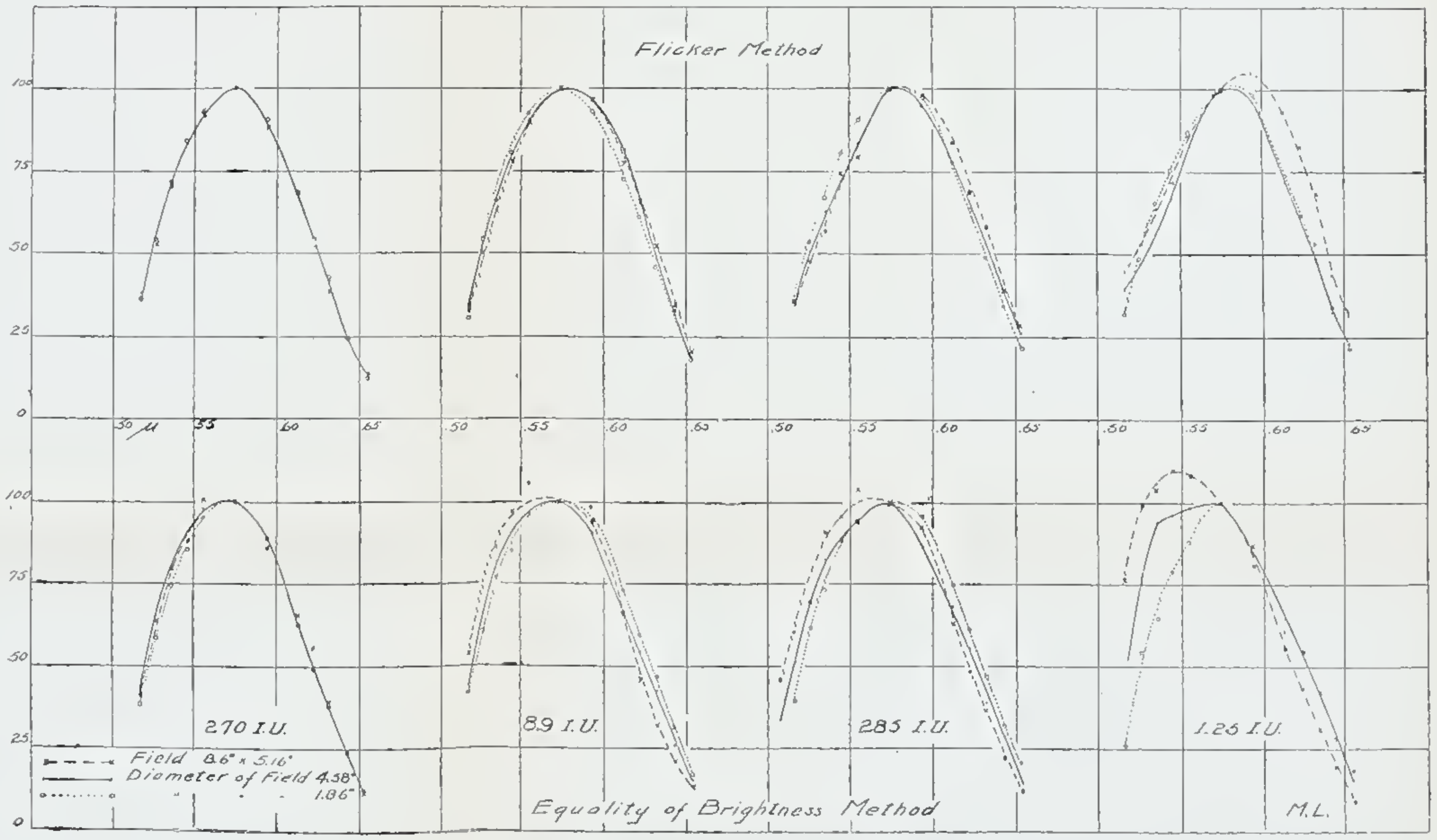


PLATE II *a*.—Effect of Changing Size of Field.

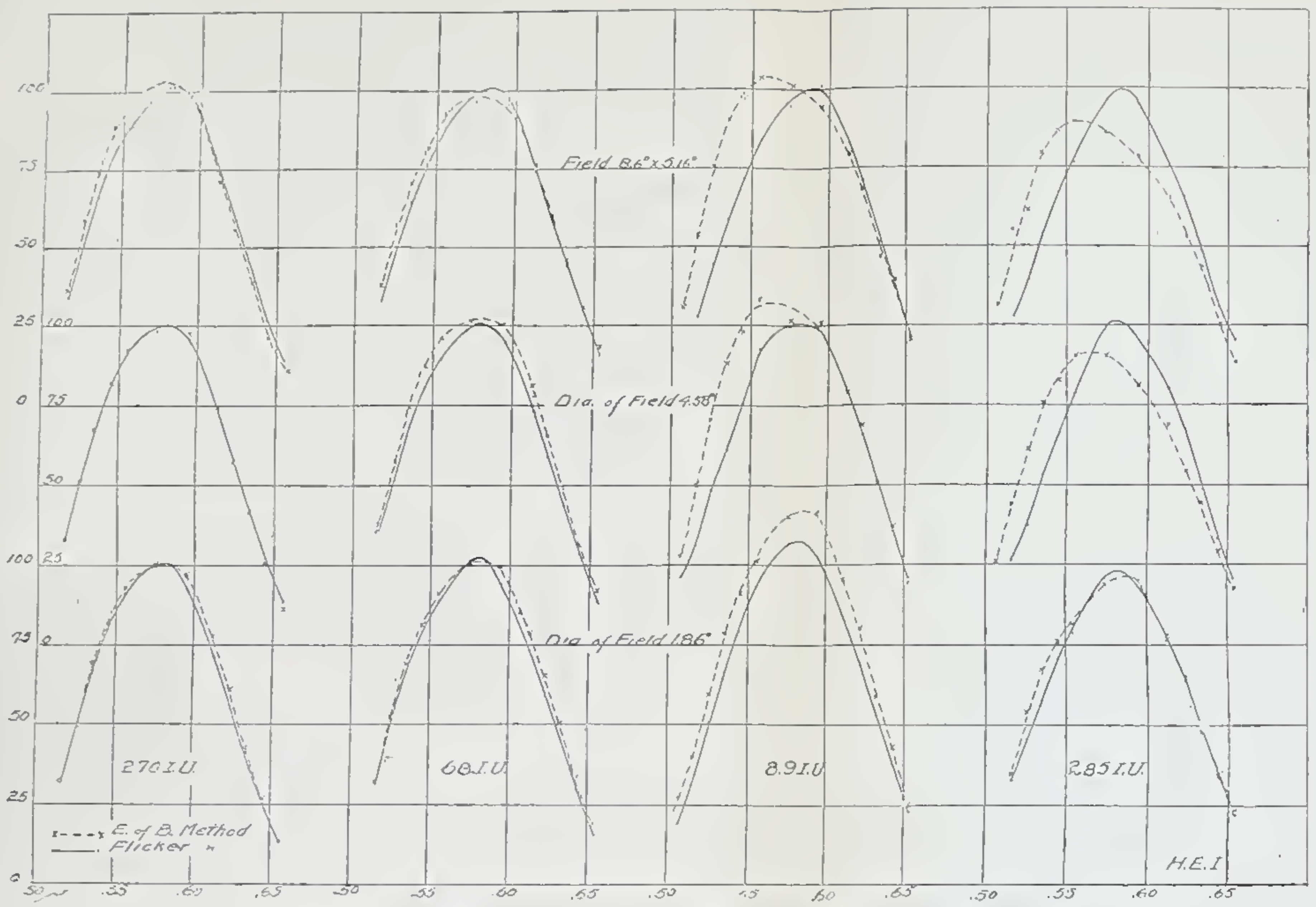


PLATE III.—Relative Position of Flicker and Equality of Brightness Curves.

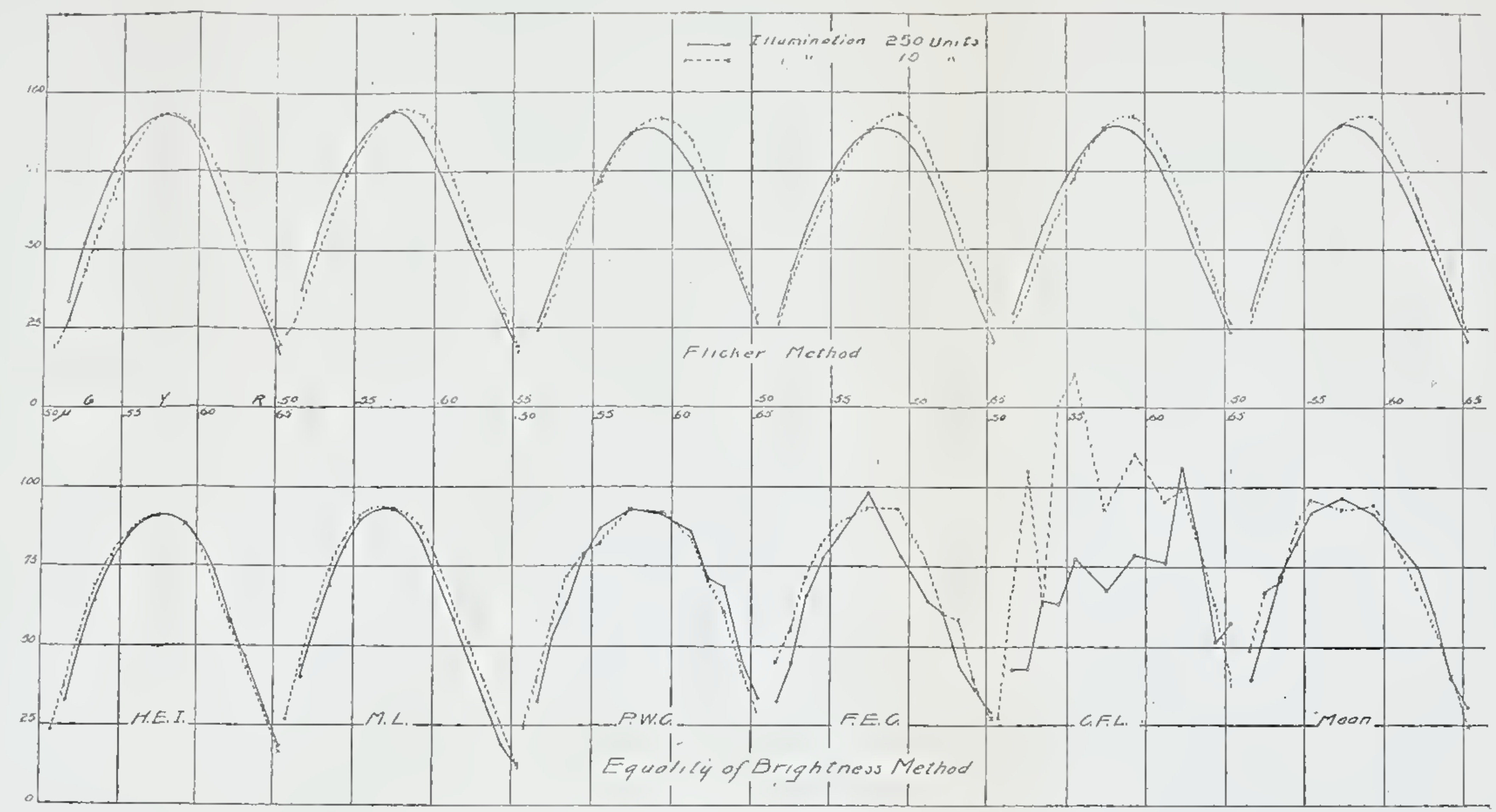


PLATE IV.—Effect of Changing the Illumination with Five Observers.

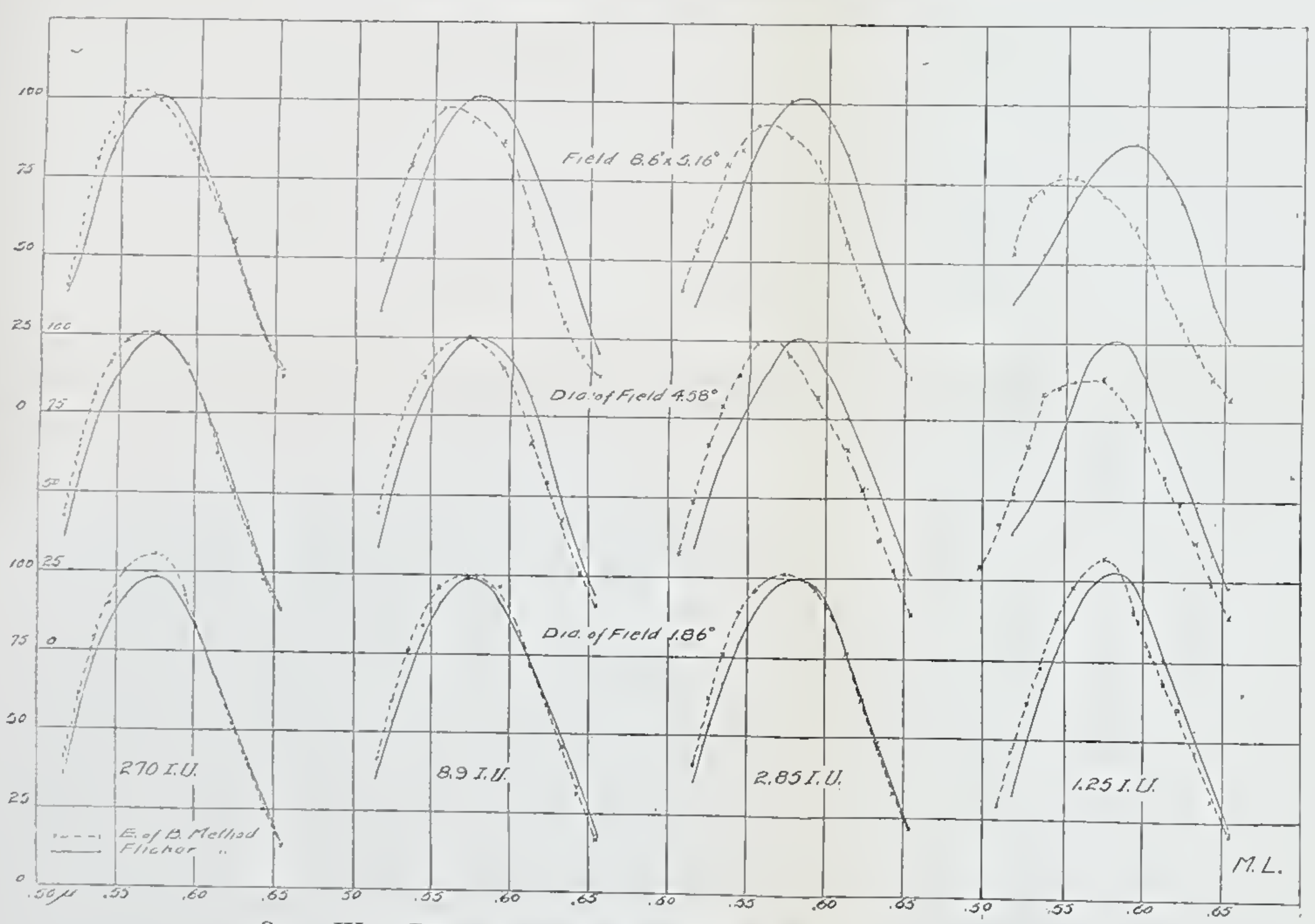


PLATE III a.—Relative Position of Flicker and Equality of Brightness Curves.

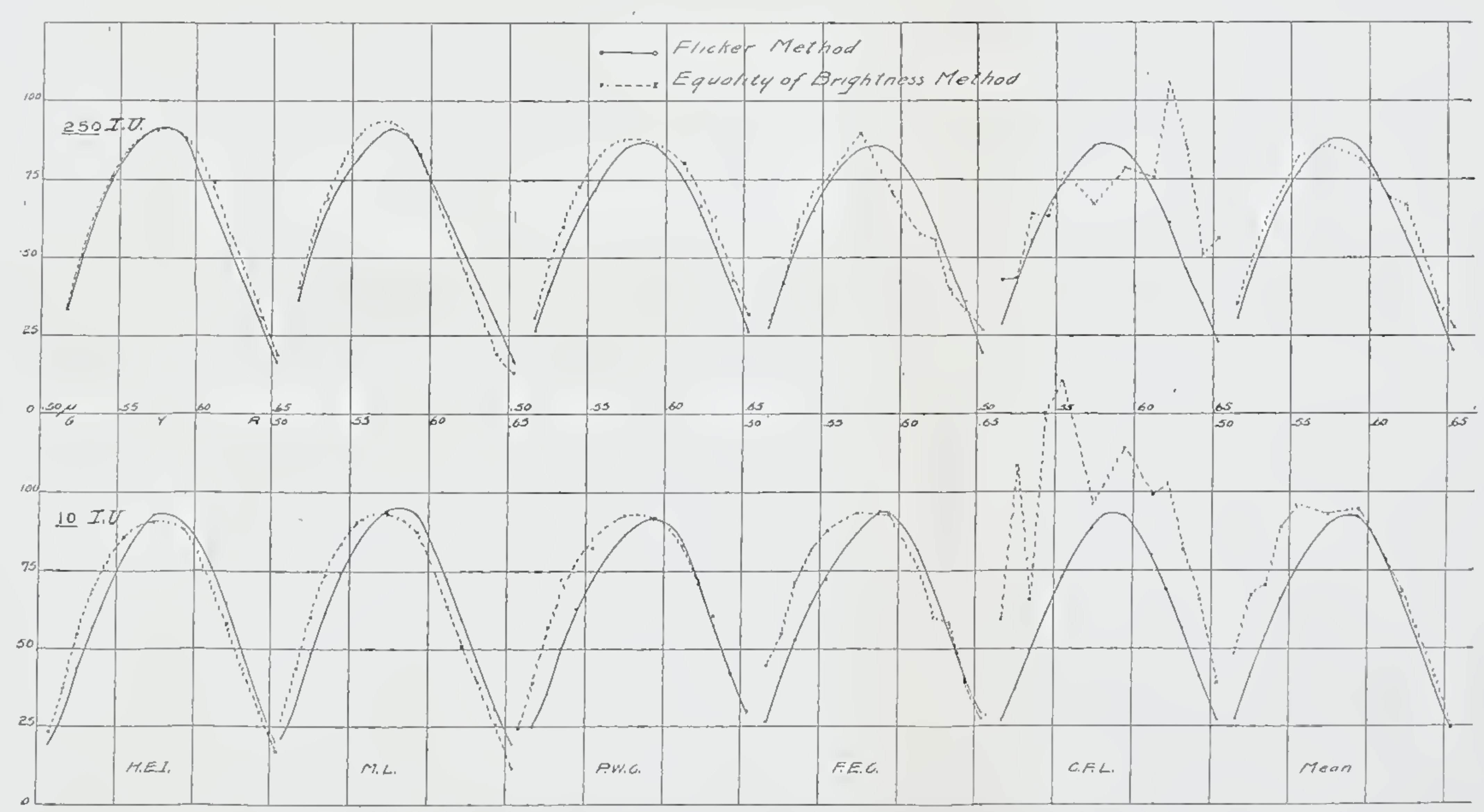


PLATE V.—Relative Positions of the Two Kinds of Curves for the Five Observers.

From the Philosophical Magazine *for September* 1912.

STUDIES IN THE PHOTOMETRY OF LIGHTS OF DIFFERENT COLOURS.

II.

SPECIAL LUMINOSITY CURVES BY THE METHOD OF CRITICAL FREQUENCY.

BY

HERBERT E. IVES.

Studies in the Photometry of Lights of Different Colours.—II. Spectral Luminosity Curves by the Method of Critical Frequency. *By* HERBERT E. IVES.

THE first paper of this series * described spectral luminosity curves obtained by the equality of brightness photometer and the flicker photometer. It was found that these curves change differently in response to variations in illumination ; the equality of brightness method exhibits the Purkinje effect, the flicker method an opposite and hitherto unobserved shift. At high illuminations the distribution of brightness in the spectrum is closely the same by the two methods, at low illuminations quite different. With the standard of comparison used—the unsaturated yellow of a standard carbon incandescent lamp—the two kinds of luminosity curves are usually of somewhat different areas at their nearest approach to similarity in shape. The differences between the two kinds of curves and the changes in each due to changed illumination are less with small fields than with large.

The present paper describes certain spectral luminosity curves obtained with the same apparatus and conditions as before but by the method of critical frequency (sometimes called "persistence of vision") †. These were studied primarily with the expectation of obtaining some light on the peculiarities of the flicker method, with which the critical frequency method is presumably closely allied. Certain seeming disagreements in the results of other observers furnished a starting point for the work here described. Stuhr, for instance, found that the method of critical frequency gave results identical to those of the flicker photometer. But Haycraft ‡ obtained, by critical frequency measurements on the spectrum, a pronounced Purkinje shift. Since, according to the writer's experiments, the flicker photometer gives a reversed Purkinje effect, it would follow that this identity of the results of the two methods cannot

* Phil. Mag. July 1912, p. 149.

† For details of each of the four methods of photometry discussed in these papers—equality of brightness, flicker, critical frequency, and visual acuity—reference should be made to the first communication.

‡ Journ. of Physiology, vol. xxi. pp. 126–146.

hold for all illuminations. Allen * has obtained "persistence of vision curves" showing a greater sensibility to pure flicker in the peripheral retina as compared with the fovea or central retina. On the other hand, the flicker photometer has been found by the writer to be less sensitive when the peripheral retina is used.

The present investigation for the most part clears up these seeming discrepancies. Most of the results of other observers have been confirmed. Their divergence is shown to be due to different experimental conditions.

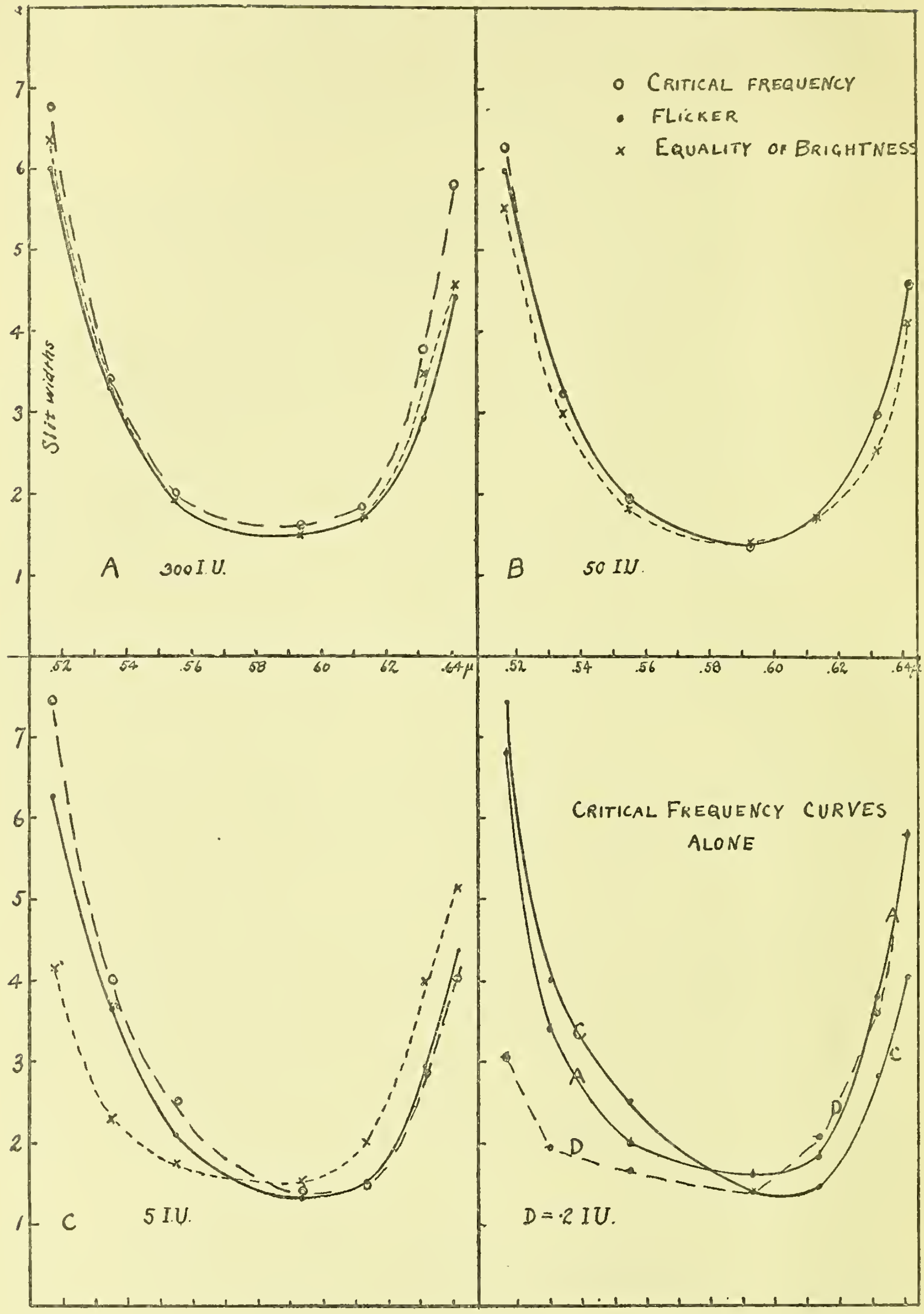
The procedure for obtaining the luminosity curves was similar to that described in the previous paper. The largest field ($5^{\circ}\cdot 2 \times 8^{\circ}\cdot 6$) was used most of the time, because with it the Purkinje and similar effects are most marked. First a set of flicker and of equality of brightness measurements was made as in the previous work. Then the spectral light was extinguished and the speed was determined at which flicker vanished when the rapidly rotating white disk was illuminated by the comparison lamp alone. The various spectral regions were then substituted for the comparison lamp, and the slit widths were determined which would cause flicker to disappear at the speed found for the comparison lamp. With critical frequency settings the mean of appearing and disappearing flicker was taken. Ten readings for each illumination were usually made, and between each set the eye was rested about a minute in the faint light of the photometer room. In this manner three kinds of spectral luminosity curves were found at each illumination. The first measurements were made at two illuminations, a medium and a low, namely 50 I. U. and 5 I. U. The results are given in fig. 1, curves B and C. (Later a variation in the procedure was found preferable as will be noted and the curves A, B, and C (fig. 1) were made by this more satisfactory method.)

It appears from these curves that the flicker and the critical frequency method both show the reversed Purkinje effect, in contrast to the regular Purkinje shift with the equality of brightness method. It appears too that the areas of the three kinds of curves differ.

The occurrence of the reversed Purkinje shift by the critical frequency method is in apparent contradiction to the work of Haycraft. It claimed first attention. For purposes of comparison Haycraft's results are reproduced in

* Phys. Rev. vol. xi. p. 257; vol. xxviii. p. 45.

Fig. 1.



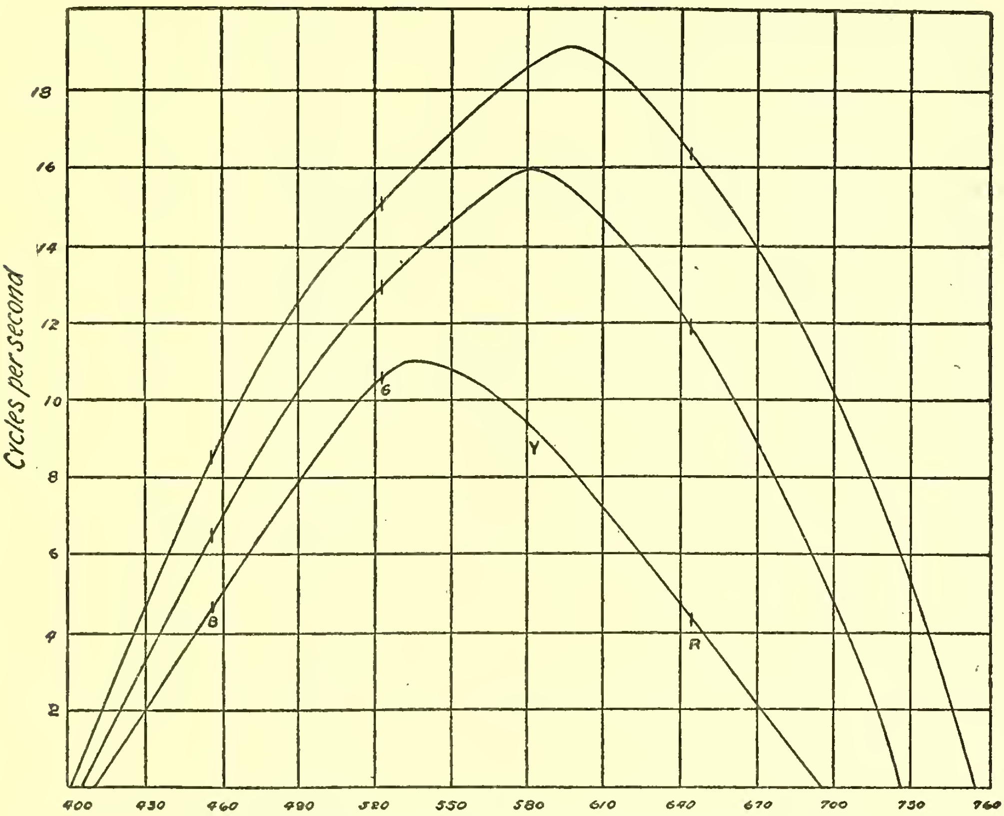
Inverse Luminosity Curves by Critical Frequency, Equality of Brightness, and Flicker Methods.

fig. 2 *. In his work Haycraft adopts a different procedure from that of the writer. The illumination is not kept constant throughout the spectrum, but is taken as it naturally occurs, consequently the ends are always much darker than

* Haycraft, *loc. cit.*

the middle. He plots critical frequencies for constant slit-width, instead of slit-widths or reciprocal slit-widths for constant frequency as is here done. It is in this difference

Fig. 2.



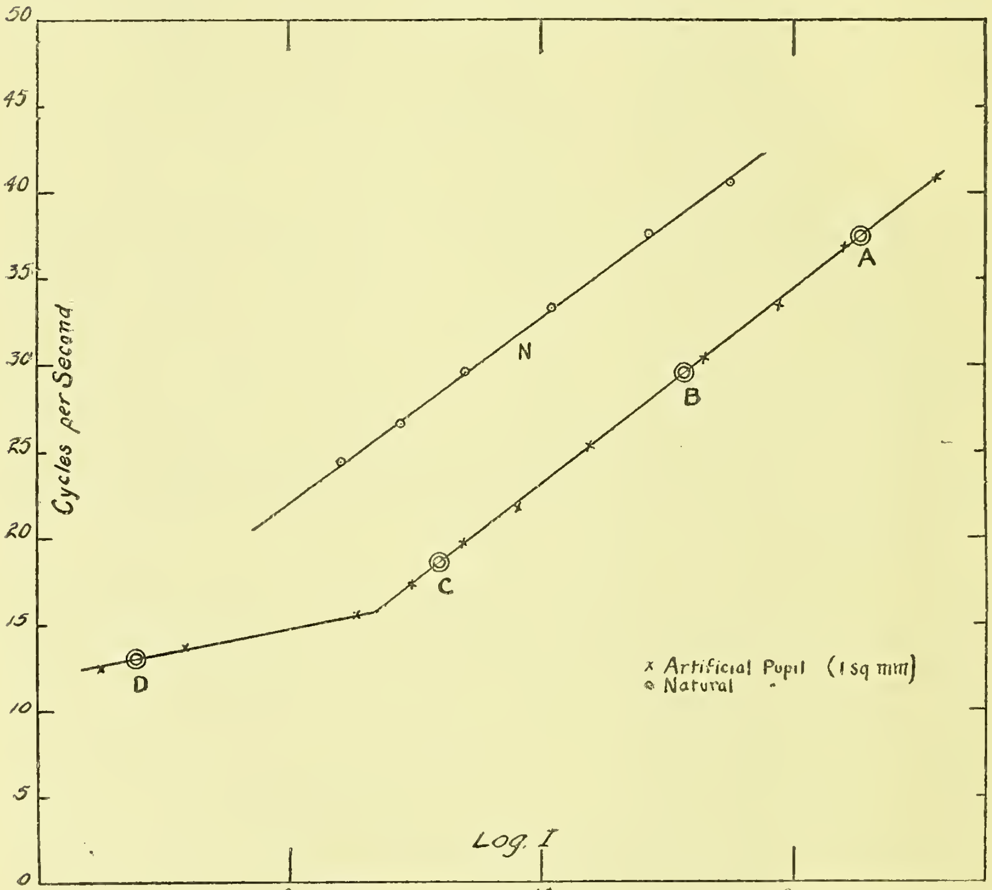
Critical Frequencies of the spectral colours at high, medium, and low intensities, showing relatively greater change for red (R). After *Haycraft.*

of procedure that we find the explanation of the difference between Haycraft's results and those of the present writer. An examination of Haycraft's curves indicates that in all cases the end portions of the spectrum are at illuminations calling for lower critical speeds than the low curve obtained by the writer. Curve C, fig. 1, corresponds to 18 cycles per second, while Haycraft's speeds vary from 19 to about 4 cycles per second. After noting this fact an additional critical frequency curve (Curve D) was determined for an illumination less than a tenth that of Curve C. This shows, with respect to the one at 5 I. U., a pronounced Purkinje shift, in qualitative agreement with Haycraft.

Apparently, therefore, the changes which accompany decreasing illumination are first in one direction and then in the other. In searching for some other phenomena of critical frequency with which this might be coordinated, it occurred to the writer that the regions of the Purkinje and the reversed Purkinje effect might be connected with the two different logarithmic rates at which critical frequency varies with illumination. Porter * has found for white light that the relationship $n=k \log. I+p$ represents the critical frequency phenomena, where $n=$ critical frequency in cycles per second, I= illumination, k and p are constants. The constant k has, however, two different values, one below ·25 M.C., another above that illumination. When plotted on semilogarithmic paper this relation is shown by two straight lines of different inclination meeting at ·25 M.C. It has been surmised that this illumination represents the point of change from cone to rod vision.

Fig. 3 gives the results of a determination of these

Fig. 3.



Critical Frequency-Illuminatiou Relations for "White" Light.

straight lines with the author's apparatus for the "white" light of the standard carbon lamp. The abscissæ are logarithms of illumination units (metre candles on magnesium oxide surface, viewed through artificial pupil of 1 sq. mm.)† ;

* Proc. Roy. Soc. lxxix. p. 313 (1902).

† The illumination is that given by the intensity of the light source divided by the square of the distance. The illumination upon the retina, when the disk is in motion, corresponds to a steady illumination of half the plotted values.

the ordinates are critical frequencies in cycles per second. The three circles B, C, and D show the illuminations at which the three critical frequency luminosity curves were obtained. The reversed Purkinje effect occurs in the upper or "cone" region, the Purkinje effect in the lower or "rod" region. In Haycraft's work the ends of the spectrum were always in the "rod" region, so that the reversed Purkinje effect would show itself only as a warping of the medium illumination parts of the curves by an amount easily to be mistaken for errors of measurement.

Before leaving consideration of fig. 3 attention is called to curve N. This was made without the artificial pupil of 1 sq. mm. aperture. It furnishes a means of evaluating the "illumination units" here used, in terms of metre candles illumination of the standard white magnesium oxide surface. To reduce illumination units to metre candles it is necessary to divide by from 7 to 8 over the range of these experiments. This figure of course applies to the writer's eye, and will be different for other eyes in proportion to the relative natural pupillary apertures of different observers. Since in working with critical frequencies the field is illuminated for only one half the time, the flicker and equality of brightness illumination units should be doubled in order to use this diagram.

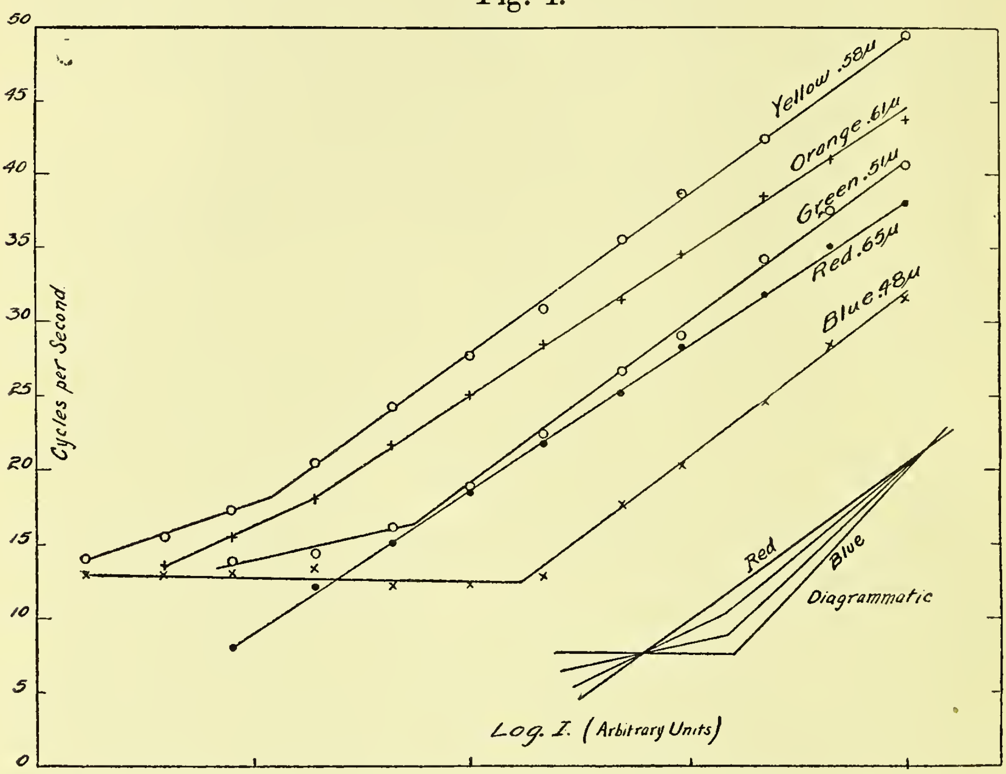
While the discovery of the relation of the Purkinje effect and its opposite to the two regions previously found by Porter is a distinct step, further insight into the phenomena follows from more detailed study. On examination of this data it is obvious that the facts presented point to different critical frequency-illumination relations for different colours. Moreover, the relation in the rod region must be different from that in the cone region, the change taking place at about 2·5 I. U.

To test this hypothesis a set of critical frequency determinations was made for various spectral colours. Illuminations were reduced by neutral tint screens * placed over the spectroscope slit. The results shown in fig. 4 bear out the surmise. In the "cone" region the different colours give straight lines (to within the errors of measurement) which are differently inclined in such manner as to give the reversed Purkinje effect. At critical frequencies of from 18–12 cycles per second these lines change their directions. Their new relative inclinations are such as to give the true Purkinje effect. The most remarkable

* "A Form of Neutral Tint Screen," Herbert E. Ives & M. Luckiesh, Phys. Review, May 1911.

curves are those for deep red (·650 μ) and for blue (·480 μ). The red curve maintains its direction. The blue curve changes from a diagonal to a horizontal line; that is, the critical frequency becomes a constant, independent of illumination. The behaviour of the other colours is intermediate.

Fig. 4.



Critical Frequency-Illumination Relations for Different Colours.

A simple relationship therefore exists between the luminosity curves at different illuminations, as given by this method. In the region above the change of direction we have, if S_λ is the slit-width at λ, F the critical frequency, $F = K_\lambda \log S_\lambda + P_\lambda$, where K_λ is a constant involving the relation between critical frequency and intensity of radiation for the individual eye for the colour in question and for the size of the photometric field, and where P_λ is a constant involving the quantity of energy emitted by the source and the dispersion and dimensions of the instrument of observation, as well as the relative sensibility of the observer's eye to flicker at different colours at a given speed. In short, the law found by Porter holds for different colours if the values of the constants are changed in the equation expressing the law. Upon determining the inverse luminosity curve or slit-width curve for any chosen critical frequency, knowledge of the constants K_λ enables one at once to construct the corresponding curves for any other critical frequency. This may be done graphically, by determining the straight lines of fig. 4 and plotting them in such units that at certain critical frequencies the values of Log. I are the logarithms of the experimentally determined slit-widths. The slit-widths for any other critical frequency (and hence illumination, by reference to the line given by the standard light source) may be read off the plot by laying a straight edge parallel to the axis of abscissæ at the desired critical frequency.

From the standpoint of the present investigation the chief interest in the study of the method of critical frequencies lies in the light it throws on the flicker photometer. For the sensibility of the method is too low for it to claim a place in accurate photometry, a matter which is discussed later. Does any simple relation similar to that just found, exist between the flicker photometer luminosity curves at different illuminations? A number of similarities between the critical frequency and the flicker photometer curves point to their close relationship. They both show the reversed Purkinje effect. The luminosity curves are at least approximately alike. Moreover, a peculiarity of the low illumination flicker curves given in the last paper, which apparently was an anomaly, is in accordance with the critical frequency phenomena. The point referred to is this: in the curve for the lowest illumination at which it was found practicable to work with the flicker photometer, the red side starts to decrease in brightness—that is, to give a Purkinje shift—while the blue side is still decreasing, thus resulting in a narrowing of the curve. Examination of the critical frequency plots of figs. 3 and 4 shows that the bend in the "white" curve occurs at a higher illumination (or critical frequency) than for blue, and at a lower illumination than for orange and red. Consequently, if the white light curve be taken as a reference standard, the change of relative direction, constituting the shift from the reversed to the true Purkinje effect, occurs sooner for red than for green or blue, precisely what was observed.

Can the flicker photometer be considered merely as a device for equating two flicker sensations which are dovetailed one into the other? The similarities just considered would suggest this possibility. We may consider two ways by which this equating could take place. Either the illumination determines the relative quantities of light which measure equal; or else the speed at which the flicker photometer is run in order to make the colour alternation just disappear determines these quantities. Both factors may play a part, and others may enter as well.

Let us consider the measurement of spectral colours against one taken as the standard of illumination, or against a white. On the critical frequency-log. I diagram, the standard and the colours to be measured are each represented by straight lines. At a given illumination a quantity of any coloured radiation may be chosen such that flicker will disappear with it at the same speed as with a chosen standard. If the rise and fall or the contour of the flicker sensation is independent of the speed or changes in the same manner for all colours with change of speed, then this coloured light may be alternated with the standard, and the speed may be reduced to the point where colour flicker is just about to appear—forming a flicker photometer. The speed not altering the equality of the two contours, the relative quantities of the standard light and the coloured will be the same as was required to make flicker disappear with each separately at the critical frequency for the standard. If this is the state of affairs the same relationship between critical frequency and slit widths exists for both standard and colour in the flicker method as with the method of critical frequency. That is

$$F = K_s \log I + p_s,$$

$$F = K_\lambda \log S_\lambda + p_\lambda.$$

We may then substitute for the critical frequency in question its value in terms of the standard illumination, and obtain as a result

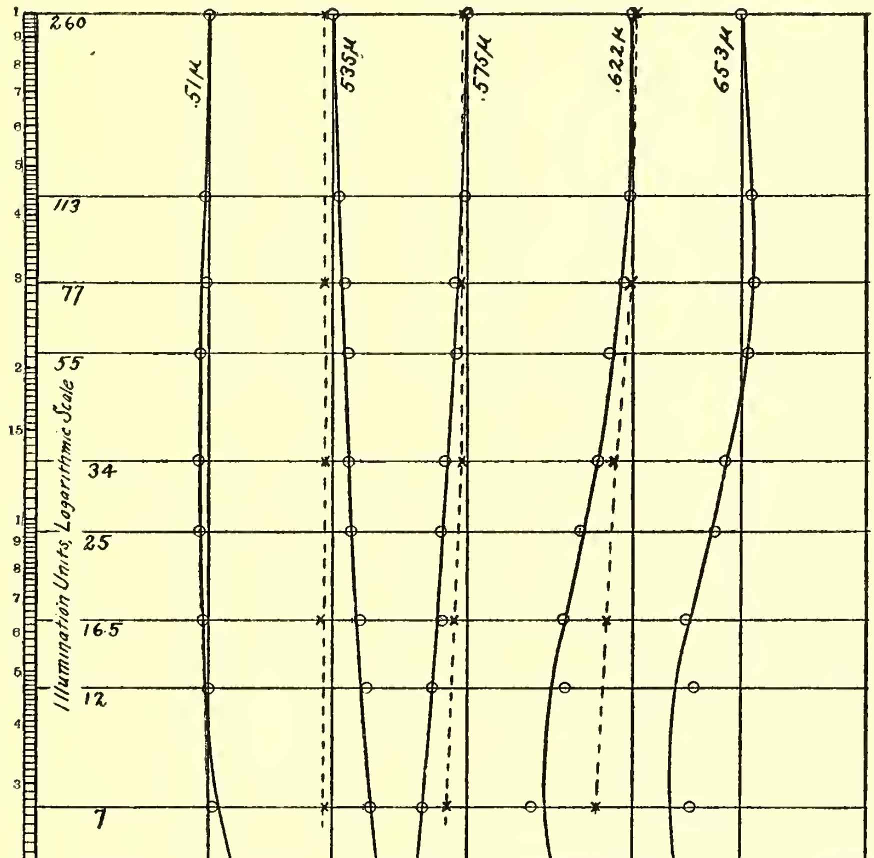
$$K_s \log I + C = K_\lambda \log S_\lambda,$$

that is, the logarithms of the slit-widths will plot as a straight line against the logarithms of the illuminations.

On the second assumption the relative quantities of the standard and the coloured radiation would be read off the diagram (fig. 4) for the speed at which the flicker photometer is run. The possibility is, however, practically ruled out by the fact that, as shown by reference to the data of the first paper, the flicker photometer speeds for an illumination showing a strong reversed Purkinje effect are 8 to 10 cycles per second; therefore, from the data in the present paper they are well down in the Purkinje region. Speed cannot therefore be the dominant factor.

Considerable time and care have been spent in testing the first possibility. The data in the previous paper were plotted in the form of $\frac{\log S_\lambda}{\log S_{.575\mu}}$ against log I. The resultant lines were so nearly straight as to raise expectations that the law of the flicker photometer had been found. To obtain a more conclusive test, isochromatics were then made. The spectral colour and the standard were matched by the flicker photometer at a series of illuminations, stepping down by means of carefully calibrated neutral tint screens. On plotting the logarithms of the slit-widths * against the logarithms of the illumination, the curves of fig. 5 were obtained. These are

Fig. 5.



Isochromatics, showing variation in relative brightness of spectral colours due to variation of illumination, Flicker Photometer.

not straight lines. The simple law anticipated does not hold. However, the effect of change of illumination is shown in a manner which indicates the method of study by isochromatics to be an excellent one for following the course

* For greater certainty slit-widths were not used in this case. Instead the voltage of the test lamp was altered, the intensity change being first determined for each colour by a spectro-photometer in which a variable sector disk took the place of the usual variable slit.

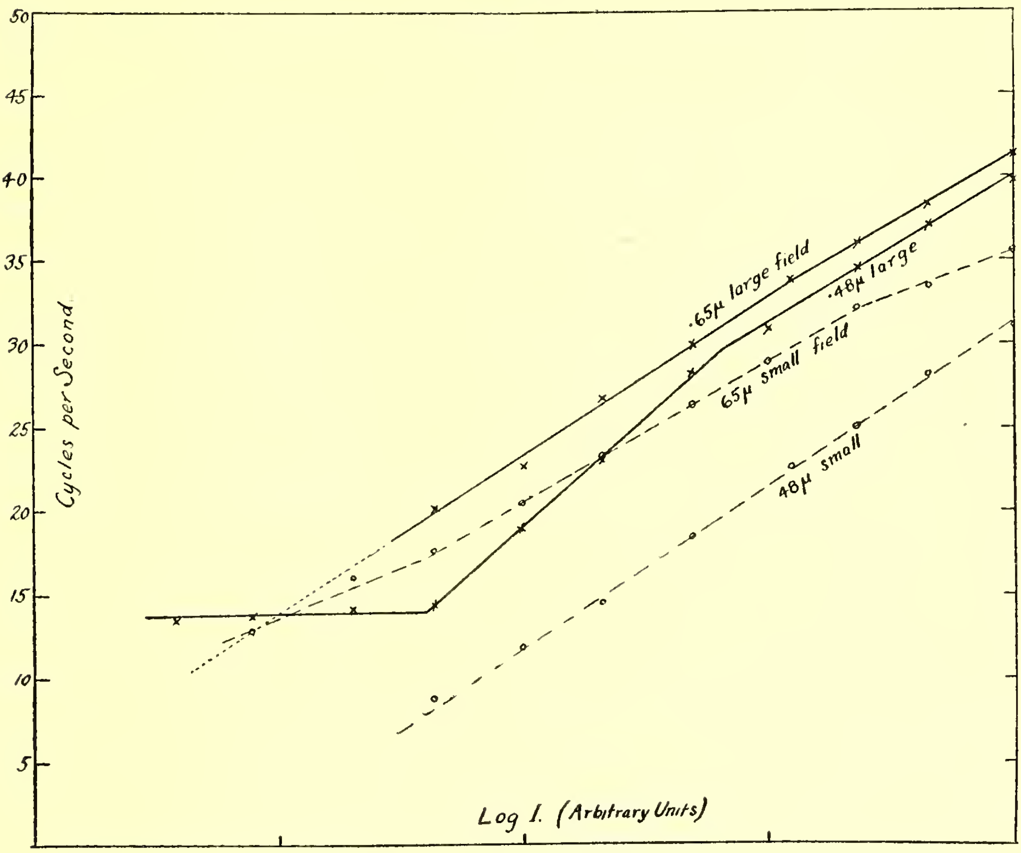
of the Purkinje and similar phenomena. It shows the rationale of changes which might, if obtained by another method, be considered within experimental errors.

The curves show the reversed Purkinje effect, making clear that the phenomena of the method of critical frequency dominate in the flicker photometer. But instead of the expected straight lines the nearest approach to them lies in the long, rather flat curves of the middle of the spectrum, while at the red and blue ends a double curvature indicates a marked deviation at high illumination from the form to be expected.

Two possible explanations of these deviations claim attention. First, that the critical frequency phenomena when plotted logarithmically are not really straight lines, but curves, and that the flicker method actually agrees with the other, but by reason of the greater accuracy of the method shows the curvature clearly. Second, that while the critical frequency phenomena are true straight lines as plotted logarithmically, the flicker phenomena are merely similar and not identical.

The first explanation is not improbable, at least for the medium and low illuminations. The deviations from straightness are probably within the errors of measurement by the critical frequency method. (The log scale of fig. 5 is many times larger than that of figs. 3 and 4.) The lowest inflexion point of the red curves corresponds fairly well to the illumination at which this should occur according to fig. 3. At higher illuminations the flicker photometer results would be explained if the separate critical frequency curves showed a tendency to bend to the horizontal, the blue showing the greatest tendency, the red the least, with white lying between. Fig. 6 shows red and blue curves, each the mean of several sets in which the eye was rested in total darkness for several minutes between each set of readings. Under these conditions the higher illuminations produce strong complementary after-images. These curves do exhibit the bending suggested and also much more for blue than for red. It may probably be explained as a fatigue or contrast effect. However, an extended series of such measurements showed that the occurrence of the change in direction is somewhat dependent on the condition of the eye, and is not nearly so easy to observe with white as with coloured light. It is also probably dependent on the character of the photometric field. The small bright area of the prism face surrounded by an entirely black field may be expected to cause contrast and induction phenomena which would be absent under less extreme conditions. It may be much more marked under the doubled illumination which holds when two colours are matched in the flicker photometer.

Fig. 6.



Critical Frequency-Illumination Relations for large and small Photometric Fields.

If the latter supposition is correct it virtually means the adoption of the second explanation, namely that the critical frequency phenomena probably follow the straight line relationship, and that the flicker photometer, influenced by various factors, merely follows this relationship closely but not exactly. In fig. 1, curve A, taken at a higher illumination than those previously mentioned, the critical frequency curve shows a larger shift toward blue than does the flicker, while in curve C the shift toward red is greater. Both these differences are near the limit of accuracy of the measurements, but are consistent in indicating that the flicker photometer method does not follow the critical frequency method absolutely. On the whole it appears more probable that the critical frequency phenomena do obey the simple law given above and that the flicker photometer phenomena merely approximate thereto. Other factors, such as speed, mutual action of different colours, perhaps the influence of the surrounding field, enter in the latter case to cause deviations from the simple conditions. At any rate the simple law which it was hoped to find does not hold for the flicker photometer.

The actual effect of this deviation from the simple law is to cause apparent constancy of the flicker curve at moderately high illuminations, since the changes over a long range of intensities are of nearly the order of the error of reading. It must not be forgotten too that these data are all for a large photometric field. Data for a small field are given in fig. 5 by the dotted lines. Their much greater constancy of value is noticeable.

A similar test of the method of equality of brightness was made by plotting both the writer's data and Kœnig's in terms of log I against log. slit-width, and some isochromatics were tried. Here again long curves were obtained. If any simple law holds with regard to these phenomena it is not this logarithmic relation, which, it may be noted, is of course merely Fechner's law.

Small Photometric Fields.

Fig. 6 shows red and blue critical frequency data for a small photometric field (diameter 1°·9). The effect of reducing the field size is to swing the blue line around nearer parallel with the red, which is swung but little. The reversed Purkinje effect will therefore be less, a condition similar to that produced in the flicker photometer by the same change of field size.

The most interesting point shown in the figure, however, is the behaviour of the blue at very low illuminations. The violent change of direction which occurs with a large field either is absent or is pushed down to a lower illumination than here tried. The fovea behaves toward blue light just as the larger area of the retina behaves toward red. This is in line with the finding of Kœnig that the fovea is blue-blind. It is here found to be not strictly blue-blind but blind to the gray sensation which replaces blue at low illuminations for a larger or peripheral field. There is also some indication that the red small field curve tends to turn at low illuminations in somewhat the manner of the blue curve for large fields. The bearing of these points on colour vision is discussed shortly.

An outstanding difference between Haycraft's work and the present research may be noted here. While his results are qualitatively explained by the data of fig. 4, these data would not give his curves. Blue light attains a fixed critical frequency of about 12 cycles per second, while Haycraft worked as low as 4 or 5 cycles per second. The explanation may be sought in two facts: his work was done with the eye in an extreme condition of dark adaptation brought on by one hour's rest in the dark before observations, and a very small photometric field was used. That the constant frequency attained by blue light depends upon the condition of the eye is shown by the different values in figs. 4 and 5, while the work on small fields shows that the turning point may be depressed by restricting the area of the retina. The combination of these two factors is probably sufficient to explain the different values of critical speeds at which the same phenomena occur in the two investigations. Peculiarities of the individual eyes, such as differences in the relative proportions of rods and cones, might also be of influence.

Sensibility of the Critical Frequency Method.

Photometry by the method of critical frequency is considerably less accurate than by either the equality of brightness or the flicker method. From among the sets of readings made near the close of the investigation one day's observations on "white" light were selected at random and studied with the following result: At a high illumination, corresponding to 43 cycles per second, the average deviation from the mean of twenty settings was 1·8 per cent. in speed; at 32 cycles per second, 1·9 per cent. in speed; at 23 cycles, 1·9 per cent. in speed. These errors appear small, but when expressed in terms of brightness instead of speed they are found to be too large for accurate photometry. They correspond respectively to 18 per cent., 14 per cent., and 10 per cent. in brightness, or five or ten times the mean error by the flicker method. Settings made on different days, or on the same day with the fatigued and unfatigued eye, are apt to differ by two to three per cent. in speed.

The critical frequency method is highly subject to disturbances whose explanation is physiological or psychological. Depending on whether the eye has been rested in diffused light, in darkness, or by blindfolding, the absolute values of the readings will be increased or decreased throughout the spectrum *. Some experiments made during the period covered by this investigation indicate that these changes may affect an unsaturated colour more than a saturated one, or less. This would cause the area of the luminosity curve to be in error.

* F. Allen, Phys. Rev. 1900 & 1909, *loc. cit.*

The slopes of the straight lines, obtained by plotting, as in fig. 3, are subject to variations, in general of such a nature as would be caused by variations in the size of the field, indicating a changing balance between rods and cones. A rather curious difference in the setting depends on whether the variable be speed or brightness. If the slit-width for any colour is held constant the critical speed may be found by several settings. If this speed is held and the brightness varied by changing the slit till flicker vanishes, the value obtained from settings for slit-width may differ as much as 15 or 20 per cent. from the previously fixed value, an effect apparently acting one way at one period and another way at another. This disturbance affects all the spectral colours to the same extent (as nearly as can be judged) so that a curve of larger or smaller area results depending on the method of setting.

In view of this fact the critical frequency curves of fig. 1, curves A to D were made by speed settings alone (except D). First the critical speed for the standard lamp illumination was determined ; then, with the slit set at the value given by the previous flicker photometer setting, the critical speed for the colour was determined. From the log I–critical speed relations previously determined the equivalent change in slit-width was read off. The relative values for the different colours, that is the shape of the curves and the Purkinje shift, were as previously obtained by the procedure described above, but these curves are given as being freer from objection.

Great care is taken in the present paper to draw only such conclusions as are warranted by the accuracy of the method.

The Areas of the Curves.

Because of the small sensibility of the critical frequency method and its susceptibility to obscure disturbing factors, it is hard to draw very definite conclusions from the difference of area shown between certain of the critical frequency curves and the flicker and equality of brightness curves. Suffice to say that such differences are apt to occur and that the previously found differences of area between the flicker and equality of brightness curves again appear *. These are sufficient to show the futility of concluding that different photometric methods give identical results simply because

* In fig. 1, curves A, the equality of brightness area is smaller than the flicker, a condition not previously found at this illumination

they show the same distribution of brightness in the spectrum. The capacity of different methods to summate will form the subject of a subsequent paper.

Peripheral versus Central Vision.

The greater sensibility of the peripheral retina to motion and to flicker has been frequently observed. In the work on the flicker photometer the possibility of using this greater peripheral sensitiveness was investigated, with the unexpected result of finding a smaller sensitiveness than for central or foveal vision. During the course of the present work frequent comparisons were made between central and peripheral sensitiveness to flicker, with different colours, and with large and small fields, at different illuminations. The findings explain satisfactorily the failure of peripheral vision with the flicker photometer.

The most significant fact was learned on the failure of several attempts to secure the critical frequency-illumination relationship by peripheral vision settings, namely, that such greater sensitiveness as the periphery possesses quickly disappears on continued gazing. It is, at least at medium and at high illuminations, merely a momentary phenomenon, not at all available for careful and continued photometric settings. The periphery, by the fatigue due to steady use, becomes for the long wave end of the spectrum, and at high illuminations, less sensitive than the centre of the retina. The fovea alone appears to have properties sufficiently constant and dependable for photometric use.

After this fact was established qualitative observations were carried out, with the eye thoroughly rested between observations. The critical speed for central vision was established, the eye was directed so that the image fell about 7° from the fovea, usually on the nasal side, and the direction of change of speed necessary to produce or destroy flicker was noted. These observations showed that for very high, nearly dazzling illumination, there is little choice between centre and periphery with red and blue light, with large or small field. A large retinal field is always more sensitive than a small one, apparently a pure area effect.

At lower illuminations the periphery becomes more and more sensitive to blue flicker, but less to red flicker. At very low illuminations (about 12 cycles per second) with a small field the difference between the behaviour of the two colours is enormous. By direct vision a blue field is almost invisible and flicker is perceived only at a low speed ; by peripheral vision the field flashes out brilliantly, and simultaneously a flicker of large amplitude appears. For red light these conditions are reversed; it is now the periphery which is less sensitive to light and flicker. This behaviour of red and blue light at low illuminations has been previously noted by Dow *.

Bearing upon Theories of Colour Vision.

Kœnig in his study of the visual purple found its absorption spectrum to correspond to the luminosity of the spectrum at very low illumination, *i. e.*, twilight or colourless vision †. The product of its decomposition, the visual yellow (which is in turn decomposed) has an absorption spectrum corresponding to the distribution of the blue sensation at high illuminations. Study of the red, green, and blue sensation curves with continually decreasing illumination ‡ shows that the colourless sensation at low illuminations arises by gradual transformation of the blue sensation—the red and green retaining their distribution substantially unaltered. At high illuminations, therefore, gray or white is a mixture of three colour sensations (using the word sensation in Kœnig's meaning), while at low illuminations the gray sensation is a simple one due to the transformed blue remaining after the red and green has dropped below the threshold. At high illuminations Kœnig considers the visual purple to be broken down successively into visual yellow and the final colourless product; at low illuminations only the first transformation takes place. Visual purple occurs only in the retinal rods, and is absent from the cones, consequently from the fovea. The fovea he finds is blue-blind.

In the first paper of this series it was suggested that the retinal cones or colour-seeing organs might be responsible for the behaviour of the flicker photometer. It was assumed that the peripheral cones, which are large, might be more red-sensitive; that is, more extreme in their cone character than the foveal cones. Situated among the rods, their exaggerated red-sensitiveness might with decreasing illumination tend to partly counterbalance the increasing part played by the rods. In the flicker photometer these cones acting alone (due in some way to the intermittent stimulus) produce the reversed Purkinje effect. Could the rods be made to act alone, an abnormally large Purkinje effect would result.

* Phil. Mag. Jan. 1910, p. 58.

† "Ueber den Menschlichen Sehpurpur," *Gesammelte Abhandlungen*, p. 338.

‡ E. Tonn, *Zeit. f. Psych. u. Phys.* vi. p. 279 (1894).

In the equality of brightness photometer (corresponding to ordinary vision) the rods and cones act together, the former prevailing at low illuminations.

The new facts learned in the present investigation seem to be in accord with the work of Kœnig and this suggested mode of action of the flicker photometer. We must add to the hypothesis, just reviewed, the assumption that the blue sensation, although produced in the visual purple of the rods, behaves in a similar manner to the red and green sensation localized in the cones. It is not necessary to assume an exaggerated sensitiveness in the peripheral rods because the greater number of rods will alone account for that. We have then at high illuminations colour vision, and response to intermittent stimuli as represented by the logarithmic relation discussed above. This state is due to cone action and visual yellow decomposition. At a certain low illumination we cease abruptly to have blue vision. In its stead we have colourless vision obeying an entirely different law with respect to flicker and ascribable to the incomplete breakdown of visual purple. In accordance with this hypothesis is the fact that the violent change of direction in the blue line of the log I–critical frequency diagram, is coincident with the change in appearance from blue to gray. Also the observation, in confirmation of the statement of Kœnig and others, that red light does not change to gray but continues red until the lowest illumination possible to make observations. The different behaviour of blue light for a small or a large field at low illumination is also in accord with the anatomical fact that the rods are missing in the fovea, and with Kœnig's observation of foveal blue blindness.

Summary.

1. Spectral luminosity curves obtained by the method of critical frequency show a reversed Purkinje effect, but at very low illuminations a true Purkinje effect, the latter observed by Haycraft.

2. A plot of critical frequencies against the logarithm of the illumination for white light gives, as found by Porter, two straight lines of different slope, which meet at about 2·5 I.U. The reversed Purkinje effect occurs above this point, the true Purkinje effect below it.

3. When separate colours are investigated and plotted in the above manner, a set of straight lines of differing slope results.

4. At about 2·5 I.U. these lines in general change their

slope, but while the line for deep red does not change, that for blue becomes horizontal, or critical frequency becomes independent of illumination. The Purkinje effect and its opposite follow at once from these facts.

5. The flicker photometer is shown to be largely influenced by the critical frequency phenomena, but not to obey the simple law which would follow were it a mere dove-tailing of two pure flickers.

6. The peripheral retina is found to be more sensitive to flicker only for momentary observation before adaptation or fatigue sets in. The fovea is more sensitive to red flicker, the periphery to blue, and the difference is more striking at low illuminations, as noticed by Dow.

7. The phenomena of critical frequency are in general accord with Kœnig's theory of the function of the visual purple and with the hypothesis that the retinal cones are chiefly active in the case of intermittent or alternating stimuli.

Physical Laboratory,
National Electric Lamp Association,
Cleveland, Ohio.
January, 1912.

From the PHILOSOPHICAL MAGAZINE *for November* 1912.

Studies in the Photometry of Lights of Different Colours.—III. Distortions in Spectral Luminosity Curves produced by Variations in the Character of the Comparison Standard and of the Surroundings of the Photometric Field. By HERBERT E. IVES.

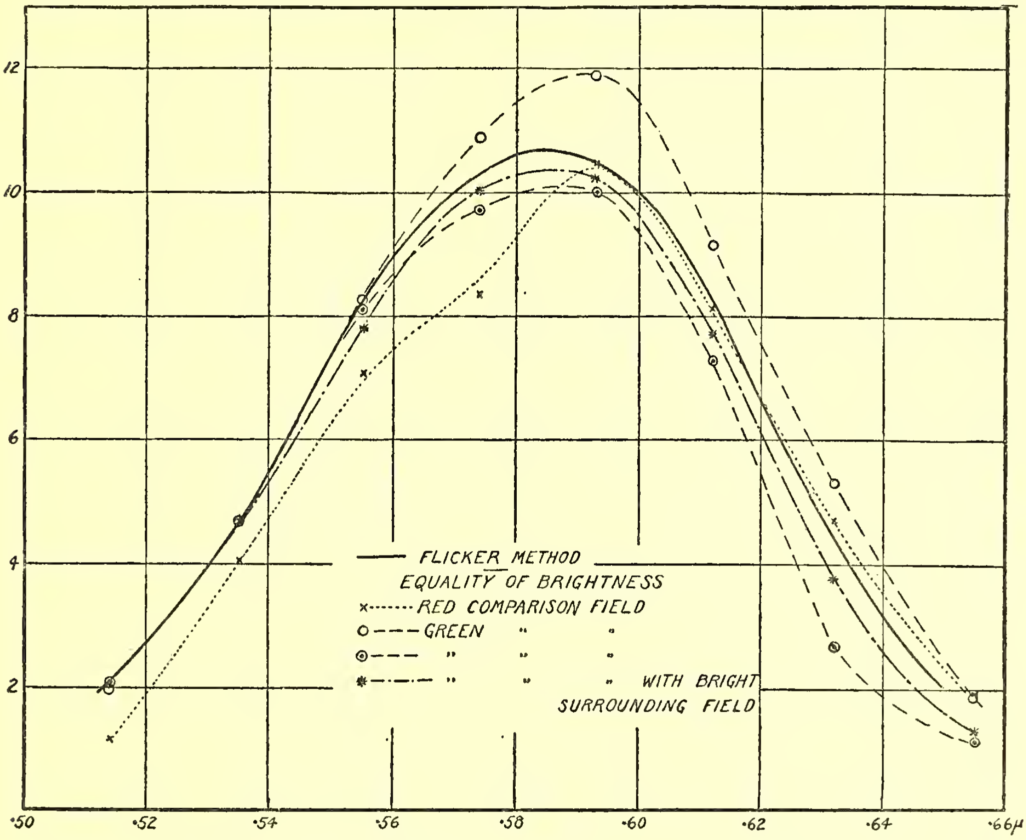
A PREVIOUS paper in this series described spectral luminosity curves obtained by the equality of brightness and flicker photometers. The mean of the high illumination values as obtained by five observers showed close agreement between the two photometric methods, although the relative positions of the equality of brightness and flicker curves varied among the different individuals. In the same paper attention was drawn to the fact that these measurements covered only a special case, namely, that of comparison of the spectral colours against a single comparison source—the unsaturated yellow of the carbon incandescent lamp. Certain experiments there quoted raised the suspicion that the luminosity curve obtained might be to some extent a function of the colour and saturation of the comparison standard due to the accentuation of colour by simultaneous contrast. As this limitation of conditions also held in the work of Koenig, who used wave-length ·535 μ as his comparison light, and of Abney who used "white," it is a matter of some interest to learn whether this suspicion is upheld or not. The experiments here described give the answer to the question at issue. In addition, another question is investigated, namely, whether an alteration in the surroundings of the photometric field, from dark to light, disturbs one's judgment of equal brightness.

In the first of these experiments no change was made in the apparatus as previously described except that red and green glasses were used in connexion with a high intensity (400 cp.) source in order to obtain red, green and "white" comparison illuminations of the same brightness. The red light corresponded in hue to ·62 μ, the green to ·55 μ. With these three colours luminosity curves of the spectrum were made under substantially the conditions of the earlier experiments. Two illuminations were used, a high one of 300 illumination units, a low one of 10 units; two field sizes were used, one of 1°·85 diameter, the other 5° × 8°; the readings were in terms of slit-widths to give equal brightness. The source measured was a normally operated tungsten lamp, as before, but since the results recorded are entirely comparative no values are given for its energy distribution, nor are corrections made for prismatic dispersion.

The results obtained were strikingly different in character by the two photometric methods. By the equality of brightness method a series of different luminosity curves were found, a different one for each colour of comparison standard. These curves vary from each other by amounts as great as would be caused by considerable differences in illumination for a single comparison standard. One of the most significant things about these curves is that no system or regularity has been found in their manner of deviating from "normal." For instance, in fig. 1 (p. 746), where are plotted a set of selected characteristic curves from this series (for high illumination), it will be seen that two made with a green comparison light at different times lie at the two extremes of character. There is not a shift in one direction for a green standard, and in the opposite for a red, but apparently any sort of shift may be expected in changing from one comparison light to another.

It is a common experience in heterochromatic photometry to form a fairly constant criterion of equal brightness when working with a definite set of colours. It is easy to imagine that such a criterion when formed must have some claim to be considered "correct." The previous work of the writer cast doubt on this belief, since the equality of brightness luminosity curves were found to change their shape and position after intervals of time sufficient to destroy one's

Fig. 1.



memory of previous settings. The present work seems to be conclusively against the belief that a "correct" criterion of equal brightness may be attained by mere practice. While with any given comparison source quite definite and consistent spectral luminosity curves may be obtained, these experiments show that after working with several different comparison sources, a return to the first one does not mean a return to the original criterion. It appears therefore that the differences introduced by changing the standard of comparison must be looked upon as psychological. One's judgment is disturbed, and the part played by judgment in the case where different colours are matched for brightness may be very large. The results plotted in fig. 1 are for high illuminations and small field; similar results were obtained for the other conditions of illumination and field size.

These remarks apply only to the equality of brightness method. The curves obtained by the flicker method showed no deviation from each other which could with certainty be considered greater than the errors of measurement. In fig. 1 therefore is plotted a single flicker curve. The equality of brightness curves are grouped about it in a manner to suggest that it is the true normal of which they are all distortions.

The second set of experiments performed was to study the effect of changing the surroundings of the photometric field. It was suggested to the writer by a psychologist that the ordinary optical instrument using a telescope subjects the eye to an abnormal condition in that the field of the instrument is surrounded by a totally black space. Under normal conditions, on the other hand, the whole retina is active. Such an abnormal condition might be expected to disturb such an unstable thing as an equality of brightness judgment.

In order to study this problem the telescope tube of the spectrometer was removed. The eye-slit was attached to a separate rigid stand, the object-glass was held in its original position by a different means. Against the objective were laid large flat metal plates pierced by openings of a size to give the same effective photometric fields as in the previous experiments. These plates were smoked with magnesium oxide and arrangements were made to illuminate them, when in place, by two incandescent lamps whose intensity could be controlled by a series resistance. The plates were of such size that upon turning on the incandescent lamps the whole field of view, except as limited by the ocular slit, was bright. This meant a field of approximately fifty degrees and was considered ample to test the matter in question.

A series of luminosity curves under the same conditions as previously obtained were made, except that both light and dark surroundings were tried. Again the results were in marked contrast as obtained by the equality of brightness and flicker methods. By the former method different curves result depending on the character of the surrounding field. But here, as before, no systematic relationship exists in the curves under the two conditions. The effect of changing the surrounding field from dark to light is to introduce a shift or distortion which may apparently be in either direction. It is again a case of disturbed judgment. In fig. 1 the effect of a light surrounding field as found in one instance is given. With the flicker method, on the other hand, changing the surrounding field introduces no variations certainly larger than the errors of setting.

Before leaving the experiments on light and dark surroundings some incidental results may be noted. With the small field, using the equality of brightness method, a very considerable increase in definiteness of setting was found with the bright surrounding field. A similar increase was not found with the large field, nor by the flicker method, but in the latter a bright surrounding field necessitated a higher speed for the rotating disk. Although no certain increase in sensibility appeared, considerably greater comfort was felt in working with the larger area of retina active, which would recommend that condition for accurate work if a small field is used.

At low illuminations with a large field where the Purkinje effect appears in the equality of brightness method, and the opposite effect by the flicker method, an interesting question is whether these effects are to be connected with the excitation of a large retinal area or only with the area which is concerned with the actual comparisons of brightness, or detection of flicker. In short, at low illuminations, will surrounding a small photometric field with a large bright area be equivalent to using a large photometric field? The answer is in the negative, for while bright surroundings shift the equality of brightness curves, the shifts are neither in direction or amount consistently such as would be called Purkinje shifts, and with the flicker method the absence of any shift disposes of the question at once.

It is evident from these experiments that no reliance may be placed upon an equality of brightness luminosity curve obtained under such conditions that the psychological element of the judgment of equality of essential inequalities is necessary. The fact that the flicker photometer is free from such uncertainty is a strong argument in its favour. There still remains unsettled, however, the question whether the flicker photometer gives something that may with propriety be called the true brightness. In other words, does it give what would be obtained by the equality of brightness method with the psychological variables eliminated?

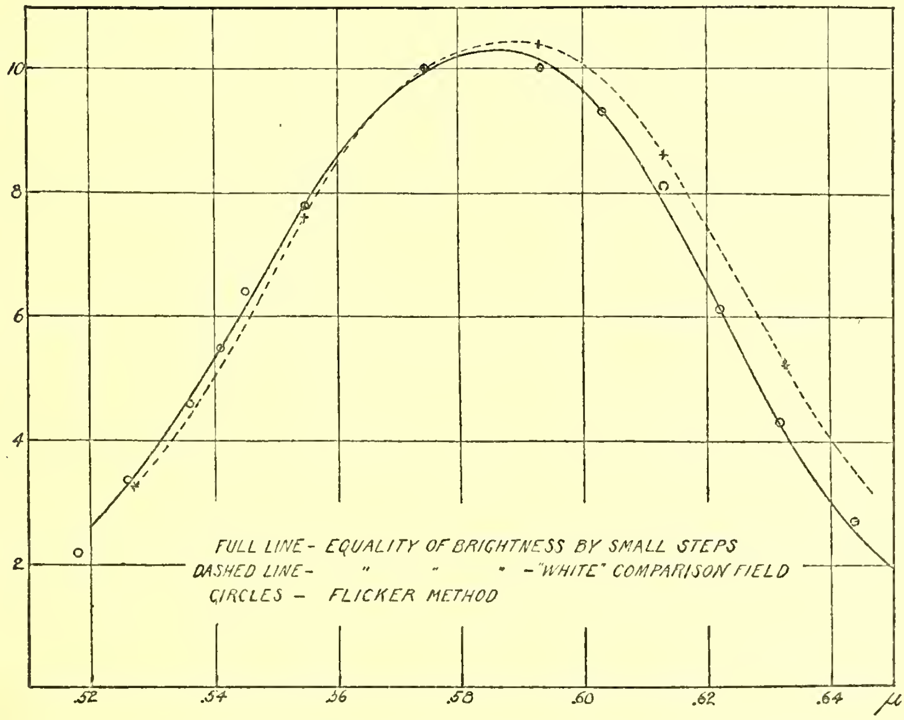
This query naturally leads to the consideration of a method of heterochromatic photometry which has been advocated and used in certain cases, namely that by steps of small hue difference, or in cascade, as it is sometimes called. The principle is that if the hue difference is made small enough it will not disturb the brightness judgment. In establishing incandescent lamp photometric standards of different colour, this method is particularly feasible, since all intermediate colours are obtainable by voltage variation.

In order to obtain spectral luminosity curves by this method, two additions were made to the apparatus as previously described. First a second spectrometer similar to the first, but with its telescope removed, was added. A first surface platinum mirror, obtained by cathode deposition on a piece of glass, which was afterward cut in two to make a sharp edge, was placed directly behind and parallel to the rotating disk, and so arranged that it could be drawn up out of place. The second spectrometer was then so placed that the light from its prism was received by the mirror and reflected into the telescope of the original instrument. In this way half the field received light from each instrument, and any two spectral colours could be compared by the equality of brightness method. The flicker disk and comparison lamp being undisturbed, it was possible at any time to secure a flicker photometer curve. As these were proved to be independent of the kind of comparison lamp used, it was not necessary to arrange to secure such curves for different spectral comparison colours, as might at first sight have been thought necessary for completeness.

These experiments were carried out only at the high illumination for the small field size, and with a bright surrounding field, these being the conditions dictated by all the work up to the present as best for accuracy and freedom from disturbances of all sorts, and so best fitted to be established as standard. Starting at the middle of the spectrum, the two sides of the field were made alike in hue and closely the same in brightness, and their relative brightness measured, then one side was changed to a wave-length only slightly removed and its brightness was measured, the wave-length difference being such as to introduce only a very slight hue difference. The comparison field was again made like the test side, the latter moved to a new wave-length and the brightness of this step compared with the first, and so on through the spectrum. Then starting at either end with the slit-width values obtained by working from the centre, the spectrum was measured continuously to the other end.

The curves obtained by this procedure were unsatisfactory, because of cumulative errors. Upon returning from red to blue, after working from blue to red, the original value for the end of the spectrum was not obtained. In order to eliminate all possibility of systematic drift such as might be caused by favouring the new hue, or one side of the photometer field, the following scheme was substituted for the first :—At numerous points in the spectrum, at the approximate slit-widths found from the mean of the previous work as necessary for equal brightness, the ratio of brightness of λ to $\lambda - \Delta\lambda$ was measured. For $\Delta\lambda$ was chosen $\cdot 004\,\mu$, which means everywhere a very small hue difference. The measurements were made at points taken absolutely haphazard, and each measurement consisted of four, first with both sides of the field illuminated by λ : second, the right-hand side by λ, the left by $\lambda-\Delta\lambda$; third, both sides by $\lambda-\Delta\lambda$; fourth, the right-hand side by $\lambda-\Delta\lambda$, the left by λ. From these two values of the ratio $\frac{\lambda}{\lambda-\Delta\lambda}$ were obtained and their mean was adopted. These values were plotted as a curve, and from this was built up the luminosity curve shown in fig. 2,

Fig. 2.



a curve which may fairly be claimed to be independent of the psychological errors illustrated by fig. 1.

After obtaining this luminosity curve, a flicker curve and an equality of brightness curve, by the ordinary method, against incandescent lamp light, were made. These are shown as well in fig. 2. It will be seen that the flicker curve (circles) agrees with the special small step equality of brightness one. The ordinary equality of brightness curve happens to be quite different from the flicker curve.

This result is in agreement with the conclusion drawn from the previous work with five observers, that when the psychological errors are eliminated the equality of brightness and the flicker photometers agree at high illuminations. In the one case an attempt was made to eliminate the psychological difficulty by taking the mean of several different observers, in the second case by a special method of observation. In the writer's opinion these results justify the opinion that the flicker photometer gives, under the conditions specified, what may be considered the true brightness.

It follows from this work that previously obtained spectral luminosity curves, such as those of Kœnig and Abney where the equality of brightness method was used, are subject to considerable uncertainties. While in the case of Kœnig's curves for different illuminations the amount and direction of the changes constituting the Purkinje effect may be substantially correct, the absolute values for the energy-luminosity values cannot be considered as sufficiently exact to be taken as normal. In view of the results of this paper it is probable that the mean flicker curve given in the first paper of the series is more entitled to be called a normal luminosity curve than any previously obtained ones.

The main results of this paper may be summarized as follows :—

1. With the equality of brightness method the effect of changing the colour of the comparison field and of substituting light for dark surroundings to the photometric field is to introduce irregular and unsystematic shifts and distortions of the spectral luminosity curves.

2. With the flicker method, the corresponding changes produce no alterations in the luminosity curves.

3. The equality of brightness spectral luminosity curve obtained by taking small steps of slight hue difference, agrees at high illuminations, for a small field, with the curve given by the flicker photometer.

4. It is concluded that the flicker photometer gives, under the specified conditions, the true brightness.

Physical Laboratory,
National Electric Lamp Assoc.,
Cleveland, Ohio.
May, 1912.

From the PHILOSOPHICAL MAGAZINE *for December* 1912.

Studies in the Photometry of Lights of Different Colours.—IV. The Addition of Luminosities of Different Colour. *By* HERBERT E. IVES.

A WELL-NIGH indispensable requirement of a method of measurement is that its renderings shall conform to two geometrical axioms. The first of these is that things equal to the same thing shall be equal to each other; the second, that the whole shall be equal to the sum of its parts.

In the majority of measurements performed in the physical laboratory the quantities dealt with are of such nature that conformity to these axioms is unquestioned. Among these measurements fall those of illuminations of identical colour and quality. But when the condition of complete identity of quality in the compared quantities is departed from, as is the case in the photometry of lights of different colour, it is unsafe to assume that these axiomatic relations hold. This paper discusses the methods of heterochromatic photometry from the standpoint of these axioms. It is shown that only one method—the flicker method—strictly conforms to the requirements in question.

As to the first requirement, that things measuring equal to the same thing shall measure equal to each other, the subject matter of the previous papers of this series gives complete information. By the methods of visual acuity and critical frequency this is so, from the nature of the measuring process, in which no real comparison source—whose character might be varied—exists. In the two remaining methods—equality of brightness and flicker—the fulfilment of this requirement reduces to this—that no change in the character of the reference standard shall affect the *relative* brightness of the different coloured lights under measurement. In the case of spectral luminosity curves this means that the shape of the curve must not be altered by a change in the reference standard. In the last preceding paper of this series it was shown that by the equality of brightness method various distortions of the spectral luminosity curves may be expected upon changing the hue of the reference standard. By the flicker method such changes do not take place. Therefore the first requirement mentioned above is met by the flicker method, but not by the equality of brightness method. Two different equality of brightness methods must, however, be distinguished, the ordinary one, where large hue differences are under comparison, and another, where by steps of small hue difference the psychological element of appraisement is eliminated. As was shown by the last paper, the latter procedure gives results identical to the flicker method. In the present paper, when equality of brightness is mentioned, the straightforward method of comparison as usually carried out with large hue differences will be meant.

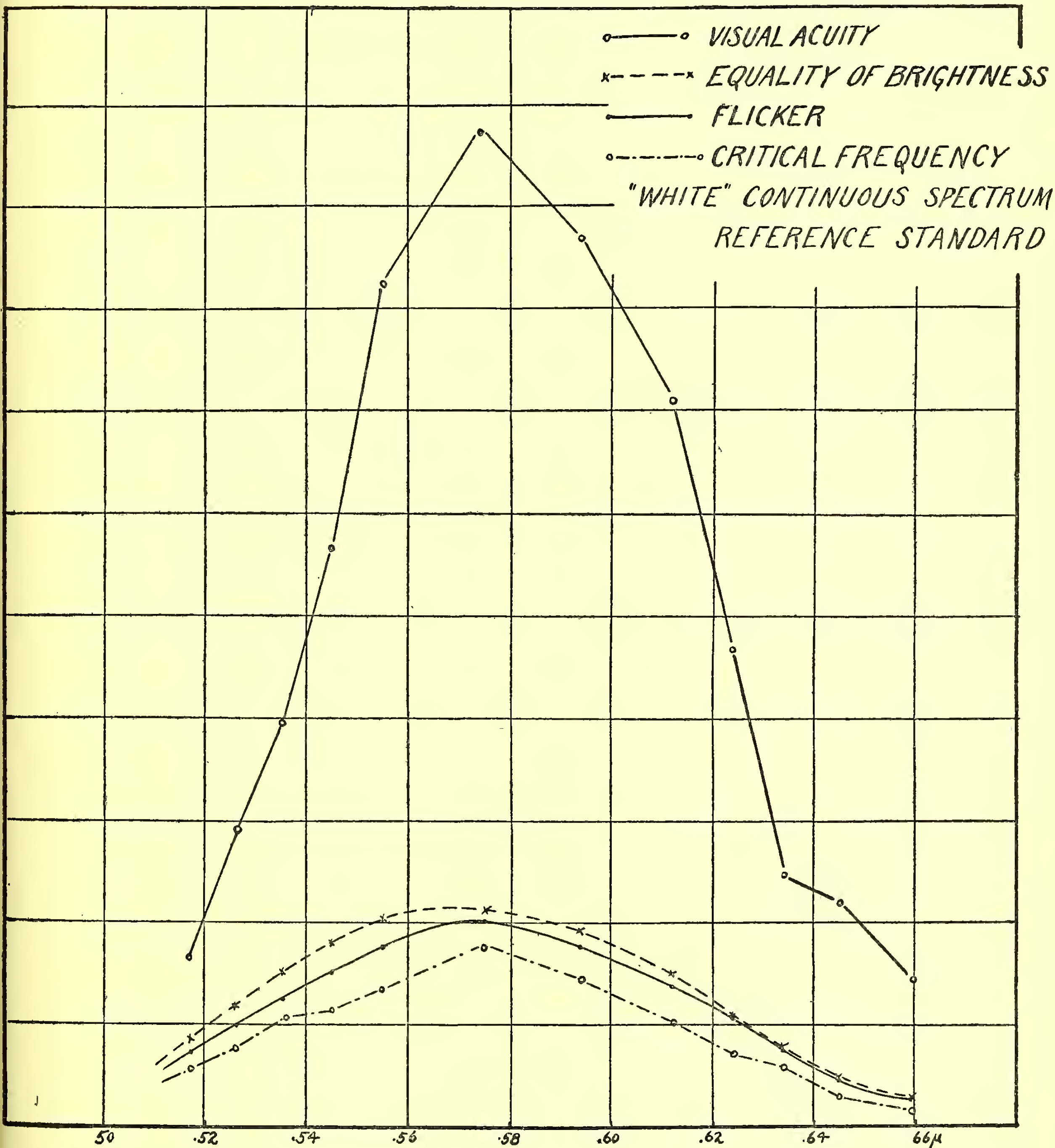
As to the second requirement—the summational one—it is to be borne in mind that while this presupposes the first one fulfilled, it may fail even with that holding. The process of physical summation may introduce factors which are not present or magnify ones not significant in the separate parts. An instance of this is given below. It is to be noted further that certain differences may exist in the results of two methods without any clash with this second requirement. Speaking in terms of spectral luminosity curves, this means that the curves by different methods may differ in the position of their maxima, and may show different directions of change with variation of illumination, but if they maintain their areas the same may all conform to the requirement in question. If, however, the areas of the luminosity curves are different, then the sum of the parts cannot equal the whole by all the methods under test. This is obvious when one imagines the luminosity curve under measurement to be that of a source identical with the comparison standard. If at the end of the spectral measurement the dispersed light is recombined, then, being identical in quality with the comparison standard, it must measure the same by all methods of measurement. If by one method of measurement this physical summation agrees with the arithmetical summation, derived through the area measurement, it cannot agree with others giving different areas.

As has been noted from time to time in these papers, curves of different area are obtained by the different methods of coloured light photometry. A striking example of this is given in fig. 1, where are four luminosity curves obtained at one sitting by the methods of visual acuity, critical frequency, equality of brightness and flicker. While they differ little in general shape and position of maxima, the enormous differences in area illustrate forcibly the error fallen into by assuming these methods identical on the basis of the similarity in shape of curves obtained by different observers. The most startling curve is the visual acuity one. This was obtained by the use of a test object consisting of a fine line grating placed over the telescope lens. Its area is five times that of its nearest neighbour, which latter differs only by a fractional part from the other two.

The enormous area of the visual acuity curve is at once explainable as due to the chromatic aberration of the eye.

Fig. 1.

Simultaneously Obtained Spectral Luminosity Curves by four Photometric Methods.



It has been shown by Bell * and by Luckiesh †, in work published since the first of these papers was written, that the defining power of the eye is much greater for monochromatic light than for complex light of the same hue. It may thus

* L. Bell, Elec. World, vol. lvii. p. 1163 (1911).

† M. Luckiesh, Elec. World, vol. lviii. p. 450 (1911).

easily happen—in the case of a purple composed of red and blue light—that increase of illumination reduces the detail-revealing power of the eye, although for the component parts of the acting light the normal relation between visual acuity and brightness holds. The visual acuity curve of fig. 1 is therefore analysable as a combination of the acuity-brightness relation and what may be termed a *quality factor.* It is a perfect example of a method of measurement in which axiom 1 is valid, but not axiom 2. Needless to say, the existence of such a quality factor effectually disposes of the method of visual acuity as a method of measuring *brightness.*

The differences between the remaining three methods are probably due to more obscure psychological causes. In the case of critical frequency the ease with which area differences may be produced has been pointed out. The manner in which the equality of brightness curves are disturbed by varying the comparison standard is sufficient reason for regarding without surprise the differences which have been found between these areas and those by the flicker method.

The crucial test of axiom 2 appears to be, in the light of all previous work, reduced to its test by the flicker method. Such tests have been made by Whitman *, and later by Tufts †. Since the latter's was more complete, the experimental test which is described below was modelled upon his, with, however, certain very essential modifications. A brief account of his experiment is necessary.

Two identical incandescent lamps were chosen, one so placed as to illuminate a white surface before the slit of a spectrometer, the other so placed as to illuminate a rotating sector in front of the telescope lens. The latter lamp was placed on a photometer track so as to be movable. The slit-width was held constant and the brightness of the spectrum measured at several points by the flicker method. At the end the *prism was removed*, and the brightness of the total light through the spectrometer was measured against the movable lamp. This was found to be within a few per cent. the same as that indicated by the area of the luminosity curve.

Further experiments showed the area of the luminosity curve to be unaltered by observing through a coloured glass, as it should not be if axiom 2 holds. When, however, a deep cobalt blue glass was used a large discrepancy was found. Certain other observers, who were less red-sensitive

* Whitman, Phys. Rev. iii. p. 241 (1896).

† Tufts, Phys. Rev. xxv. p. 433 (1907).

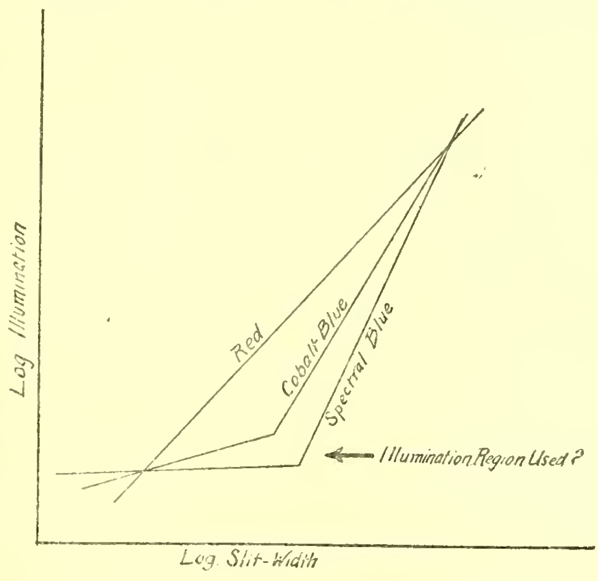
than Tufts, showed a discrepancy in the same direction even with white light.

The points to which attention should be directed here are, 1st, that the conditions of illumination and field-size are not stated, nor was the illumination constant. Instead of taking portions of the spectrum of equal brightness, equal slit-widths were taken, with consequent low illumination at the red and blue ends. 2nd, there is no mention of correction for the light lost by absorption and reflexion in the prism. It is hard to imagine that this correction was not made, but the test is inconclusive as long as this is in doubt.

With regard to the peculiar discrepancy with the deep cobalt glass, this can be explained, qualitatively at least, by the results of paper II. of this series. Bearing in mind that the effect of this screen over the eye is to reduce the illumination excessively, it is quite possible that the spectral blue measurements fell in the region indicated by the arrow in the (slit-width) diagram sketched in fig. 2, in a region where

Fig. 2.

Suggested Explanation of Anomalous Results of Tufts.



the law of the flicker photometer changes (see paper II. of this series). The result of this would be the effect found by Tufts. However, these low illumination effects, interesting as they are in connexion with the theory of colour vision, cannot be held of primary interest in practical photometry.

A repetition of this experiment is here described, in which the undesirable features of Tufts' experiments are eliminated. Because this work is intended primarily for application to practical photometry no repetition is made of the low illumination tests, nor tests on observers of abnormal colour vision.

The apparatus used is that employed in the previous investigations described in this series with certain modifications. An important feature is a special slit, made with both jaws movable. One is moved by the divided head as before (adjustable edge) ; the other is free and may be brought into contact with the first by merely sliding in its ways. The method of use is to leave the prism fixed throughout the experiment ; the adjustable edge of the slit is placed in a position corresponding to the ultra-violet, the free edge brought into contact with it. The adjustable edge is then moved by the divided head until, by the flicker criterion, a piece of the spectrum is transmitted equal in brightness to the comparison standard. The free edge is then brought up into contact with the adjustable edge and another piece of the spectrum of the same brightness exposed by moving the latter. In addition to this slit another means of varying the amount of spectral light is furnished by a variable neutral tint screen * between the light source and the ground glass over the slit. This is used at the end of the spectrum, where the last portion of spectral light consists of the whole remainder of the spectrum and cannot be varied in brightness by slit-width variation. (This screen can of course be used at any point if so desired in place of slit-width variation, it being only necessary to take slit-widths of suitable size so as to give approximately the same amount of light.) At the end of the spectral measurements the free jaw is drawn back to its original position, whereupon the eye-slit receives the whole spectrum which it has before received in parts. The prism used in this test was a special constant dispersion, constant deviation type, made by A. Hilger. It gives a considerably shorter spectrum than that furnished by the regular crown glass prism used before.

A new form of flicker photometer was used in place of the rotating disk of the previous work. This consists of a first surface metal mirror on an oscillating lever. The mirror was made by coating a glass plate with platinum by cathode discharge, then cutting the glass with a diamond. The edge where the glass breaks is almost invisible in the photometer field †. The other end of the lever is driven by a link motion attached to the speed counter axis and having a 1-inch stroke.

This mirror device gives a close approximation to a perfect flicker photometer. With regard to the question of

* "A Variable Absorption Screen," Ives, Elec. World, March 16, 1912.

† Pfund, Johns Hopkins Univ. Circular, vol. iv. 1906, pp. 20–22.

perfection in flicker photometers, a word may be said. Some experimenters have thought they found differences in the results with the flicker photometer when it was operated at different speeds. As a matter of fact there is only one speed for such a photometer, namely, that which just causes colour flicker to disappear. Any greater speed causes a decrease of sensibility. Now as the colour difference is decreased a decrease in speed and an increase in sensibility are made possible. If there is no difference in colour, the speed drops to *zero*. In other words, *for lights of the same colour the flicker photometer approaches as its limiting case the equality photometer.* In the instrument now described this is proved to be literally so. By proper choice of voltages on the test and comparison lamp, and by slightly tinging the light from the latter by letting it fall upon a flashed opal glass over which is laid four thicknesses of lantern-slide cover-glass, the total spectral light, obtained from the wide slit, and the comparison standard light, are identical in appearance. The speed for greatest sensibility is as it should be, *zero*.

Another detail of some importance is that in measuring the total light through the prism a neutral tint screen * is placed over the slit, of such transmission that it reduces this light to approximately the same brightness as the comparison standard, the exact setting being made by means of the variable neutral tint screen before mentioned. The object of this is to have all the measurements carried on at the same brightness, although since this final measurement is of lights of the same colour it is not perhaps absolutely necessary.

It may fairly be claimed for this arrangement of apparatus that physically the whole is exactly the sum of the parts. There is no measurement of areas ; all scattered light is present exactly in the same manner in the fractional and total light measurements ; there are no instrumental changes for which correction need be made, such as the removal of the prism in Tufts' experiment.

Three sets of measurements were made. For each an illumination of 300 I.U. held ; the field was 2° diameter, and surrounded by a large bright area (25° diameter). Two sets were made by the writer, whose luminosity curve lies nearly at the blue extreme of the observers previously examined, one set by Dr. P. W. Cobb whose luminosity curve lies at the red extreme.

* "A Form of Neutral Tint Absorbing Screen," H. E. Ives & M. Luckiesh, Phys. Rev. May 1911.

The results were as follows :—

	By parts.	Total.
H.E.I. I..........	691	695
H.E.I. II..........	742	743
P.W.C. I..........	773	785

(Slight changes in the position of the variable screen and comparison lamp cause the different magnitude of the results in the arbitrary units given.) The greatest difference is in the case of the less practised observer and amounts to $1\frac{1}{2}$ per cent. This lies within the range of his errors of setting.

The conclusion is drawn that under the conditions of high illumination and small field size specified, the arithmetical sum of the parts as measured agrees with the measured value of the whole, by the flicker method, for observers of normal colour vision.

With the proof of this property of the flicker method of photometry is concluded the set of tests which were planned at the beginning of the investigation. A general summary of the characteristics of the flicker method follows :—

1. It surpasses all other photometric methods in sensibility and reproducibility in the presence of colour difference.

2. It agrees at high illuminations with the equality of brightness method, when the latter is freed from the psychological uncertainties inherent in its use.

3. It measures at high illuminations what may fairly be termed the true brightness.

4. Brightnesses measuring equal to the same measure equal to each other and the sum of the measurements of the parts is equal to the measurement of the whole.

In the opinion of the writer the flicker photometer should be adopted as the standard instrument for making heterochromatic comparisons in the standardizing laboratory. Before, however, this can be done, the standard conditions of illumination and field-size should be agreed upon, and, most important of all, the normal luminosity curve of the eye should be determined from observations on a large number of individuals. This done, not only will it be possible to determine which individuals should be chosen to make such comparisons, but also, in virtue of the various qualities of the flicker photometer, it will be possible to correct to normal the results of any observer whose luminosity curve is known. The next paper of this series will be devoted to a discussion of standard conditions for heterochromatic photometry and give values for the luminosity curve of the average eye.

Attention may here be called to the fact that the solution of the problem of heterochromatic photometry clears the way for the establishment of a rational primary standard of light flux, namely, a certain rate of flow of radiant energy of maximum luminous efficiency*. It is hoped to use the results of this study of coloured light photometry in a determination of the luminous equivalent of the most efficient radiation, a quantity which, in accordance with the above suggestion, is identified with the rational primary standard of light.

Physical Laboratory,
National Electric Lamp Association,
Cleveland, Ohio.
June 1912.

* "Energy Standards of Luminous Intensity," H. E. Ives, Trans. Ill. Eng. Soc. March 1911; "Luminous Efficiency," Elec. World, June 15, 1911.

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Studies in the Photometry of Lights of Different Colours.— V. The Spectral Luminosity Curve of the Average Eye.

By HERBERT E. IVES.

ACCORDING to the results published in previous papers of this series the flicker photometer, under certain conditions, conforms most nearly of all photometric methods to the requirements for a method of heterochromatic photometry. Granting its adoption as the standard instrument for this kind of photometry, there remain to be determined the characteristics of the average human eye. Given the satisfactory photometric method, and the standard eye to use it, the measurement of the relative brightness of differently coloured lights becomes a definite thing.

The present paper gives the result of an experimental determination of the mean luminosity curve of eighteen observers, sixteen men and two women, aged from 18 to 40 years. Since none of these were colour-blind or possessed any known abnormalities of vision, the mean of their values may be considered as that of an average normal eye. The number of observers is sufficiently great to warrant the belief that the mean thus obtained is sufficiently near the absolute mean eye for all practical purposes.

The conditions holding during the measurements, the reason for their choice, the details of the work, together with the results, are discussed below under the several headings.

Photometric Conditions.

Flicker Photometer.—The flicker photometer was chosen for reasons made clear in the previous papers of this series. It possesses the greatest sensibility, its results are most reproducible; by its use, things measured equal to the same thing measure equal to each other, and the sum of the measurements of the parts equals the measurement of the whole.

Illumination.—An illumination of 25 metre candles was chosen since, by the means described below, this was found to correspond closely to the high illumination at which the results of the flicker and equality of brightness methods become the same. This is a practicable illumination for the usual laboratory light sources and photometers; it corresponds to a comfortable working illumination for reading, &c., and the changes in visual colour sensitiveness on going to the still higher illuminations common by daylight are probably small, perhaps negligibly so.

Size of Photometric Field.—A photometric field of 2° diameter was selected; that is, the smallest field used in the previous work, for the reason that the changes in the luminosity curve due to changes in illumination are least with the small field. This field size, together with the high order of illumination chosen, would permit considerable latitude in the illumination without appreciable effect on the result.

Bright Surrounding Field.—The small photometric field was surrounded by a bright field of about 25° diameter, maintained at approximately the same brightness as the photometric field. This bright area, although previously not found to affect the results of the flicker photometer to an extent greater than the errors of measurement, was used for the greater comfort which it gives when observing with a small field. It is not an essential part of the scheme of measurement, but for the sake of complete uniformity should be used always, if at all.

Apparatus.

The apparatus was the same as in the previous work. A Hilger spectrometer with a $\frac{1}{2} \times 2$ mm. slit in place of the eyepiece; the light from the comparison lamp (4 watt carbon), after falling upon a magnesium oxide surface, is reflected into the spectrometer telescope from a first surface platinized glass mirror arranged to oscillate and so form a flicker photometer; the diaphragm which limits the size of the field is white and is illuminated by auxiliary carbon lamps. The speed counter, slit mechanism, &c., have previously been described in detail. The light source whose spectrum was measured was, as before, a tungsten lamp; the details of its use are given below.

Details of Measurements.

Energy Distribution in Spectrum of Light Source.—The spectral luminosity curve which is desired is that of a known energy distribution, from which the luminosity curve of any other distribution may be derived. For convenience it is desirable to give the results for an equal energy spectrum. Were it feasible to measure the intensity of radiation directly at the eye-slit this would be the ideal measurement, since it would at once eliminate the determination of various instrument corrections. In the present investigation this was not considered practicable, so measurements were made on a set of four frosted bulb tungsten lamps whose energy distribution was determined as follows:—All four lamps (previously seasoned) were carefully colour-matched on a photometer against a tungsten standard of 1·15 watts per mean horizontal candle-power. Immediately before and immediately after the use of each lamp on the flicker spectrometer it was measured on a spectrophotometer (substitution method) against a tungsten lamp which had previously been measured against a black body of known temperature *. This latter lamp had a visual energy distribution corresponding to a black body at 2295° abs. The frosted lamps were compared with it at four points in the spectrum. The distribution for a black body,

$$J_\lambda = c_1 \lambda^{-5} e^{\frac{14500}{\lambda T}},$$

was plotted as a straight line in the form

$$\log \frac{J_\lambda}{c\lambda^{-5}} = \frac{14500}{\lambda T} \cdot \log_{10} e.$$

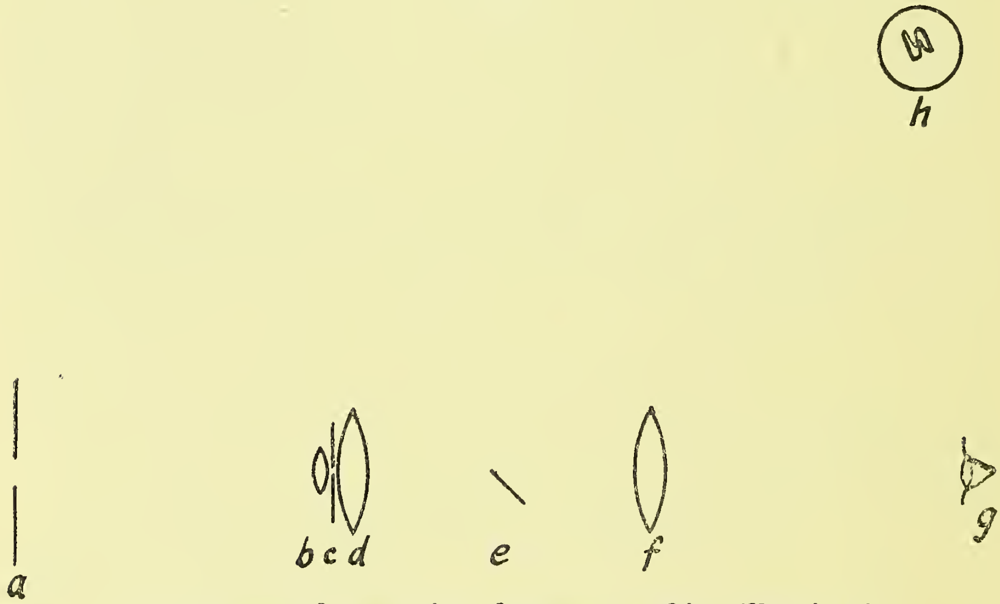
The spectrophotometer ratios were plotted (in logarithmic units) against this straight line; through these points another straight line could be drawn if the new energy distribution corresponded to another black body colour, as is pretty well established. This was found to be the case, and from the slope of this line the equivalent black body

* This standard of black body distribution was obtained and has been maintained for this Laboratory by Dr. E. P. Hyde.

temperature of 2360° abs. was found for the lamp in question. The distribution calculated for this temperature was in turn corrected for the selective absorption of the prism, ground glass, and lenses, determined spectrophotometrically, as well as for the dispersion of the prism. Applying the final value of the energy distribution to the readings, the results are obtained for an equal energy normal spectrum.

Elimination of Artificial Pupil.—In all the previous work an artificial pupil was used and the results were given in terms of metre-candles illumination as viewed through this 1 sq. mm. aperture. In working with a spectrometer the use of a small eye-slit is practically imperative. But in practical photometry an artificial pupil of this size would necessitate working at illuminations too high to be practicable with present illuminants if one had to attain the retinal illumination called for by the investigation here described. Were the pupils of all observers of the same size under the same conditions, a reduction factor might be obtained so that the luminosity curve could be found with the artificial pupil and used for a corresponding illumination with the natural pupil. Such, however, is not the case.

Fig. 1.



Arrangement for securing the same working illumination for all observers.

In view of these facts it was considered advisable in the present research to have all curves made for a normal pupil illumination. In order to accomplish this, the arrangement shown in fig. 1 was adopted. *a* is the diaphragm limiting the photometric field ; *b* is a small spectacle lens of focal length *ab* ; *c* is the eye-slit or artificial pupil ; *d* is a lens of focal length *de* ; *e* is a magnesium oxide surface illuminated by the lamp *h* ; *f* is a lens of focal length *ef*. At *g* is formed an image of the slit *c* ; the eye placed here sees the image of the diaphragm *a* in the same plane as the edge of *e*, and by moving the comparison lamp the field may be made of the brightness of *e*. The auxiliary lenses are then removed, proper correction made for the loss of light caused by the various lenses, and the slit *c* used as before.

A test of the illumination used in the previous investigation of the series—which in turn had been chosen to be in the region where the flicker and equality of brightness method had been found to agree, and where large and small field agree, viz. 300 "illumination units"—showed that for the writer's eye it was close to 25 metre candles. As this was desirable from various standpoints, it was taken as the constant illumination on *e*. Each observer was required to find the artificial pupil illumination corresponding to 25 metre candles normal pupil, and his luminosity curve was then obtained at that illumination. In this way the results all apply to the standard illumination under ordinary photometric conditions. It was found that the highest illumination used was about twice the lowest. This range would make no noticeable difference in the luminosity curve with the illumination and field size used.

Details of Procedure.—The measurements of brightness were carried out as in the previous work, by finding the slit openings to match a constant comparison illumination. The zero of the slit was found as follows :—By means of colour screens the light of the comparison lamp was made to match a certain wave-length ($\cdot 59\,\mu$) ; then a series of three neutral tint screens of known transmission were placed successively over the comparison lamp side and the slit width obtained for photometric match. These values when plotted against the transmissions give a straight line passing through the effective zero. This method proved very satisfactory.

The calibration for wave-length was provided for by having the eye-slit adjustable sideways by a milled head ; a helium tube was placed behind the tungsten lamp under measurement so that it could be turned on at intervals during the measurements and the setting of the drum checked. This prevented any instrumental drift.

A difficulty was met with in carrying the measurements down into the blue of the spectrum, namely, that the necessary slit widths became altogether too great for sensibility or accuracy. This difficulty was met by a compromise. For the measurements at $\cdot 50\,\mu$ and $\cdot 48\,\mu$ neutral tint screens of $\frac{1}{3}$ and $\frac{1}{5}$ transmission were used, so that these measurements were actually made at lower illuminations than the others. Since, however, the small field was used, and since even these illuminations are only just getting into the region of rapid change of luminosity curve shape (with large field), the errors will be small. They are also in just the opposite direction from the small errors due to the use of a wide slit in this spectral region.

The manner of taking observations was to start at the middle of the spectrum, then take observations back and forth, from red side to blue side, working toward the ends of the spectrum. This was found a desirable procedure, because it gave the observers a chance to obtain practice on the measurements with least colour-difference. At the end the first two or three readings were repeated until a close check was found. With some observers a check was obtained on the first wave-length, while with others there was a difference of several per cent. until the second or third wave-length. The mean of both sets was taken where a difference was found. Such differences are probably to be ascribed to lack of practice at the start.

All the scale-readings were made by an assistant, who watched voltages and held the speed constant.

Observers.—Of the eighteen observers, the majority had never used a flicker-photometer, while several had never before looked in a photometer of any kind. It is a striking point in favour of the flicker-photometer that its method of use can be acquired so quickly that there is little choice between the results of skilled and unskilled observers. The former differ from the latter chiefly in the greater speed with which they can make their readings. In instructing the inexperienced observers the procedure was compared to placing a blind man upon the side of a hill and asking him to find the top. As he finds his position relative to the summit only by stepping off in various directions, so the photometrist locates the minimum of flicker only by oscillating the illumination (slit) until each direction of motion increases the flicker. Standing still in either case is useless. The correct speed was determined as that which made the peak of the "hill" the sharpest.

Results.

The readings were first reduced to a normal equal energy spectrum as described above. The resultant curves were measured for area with a planimeter; then all were reduced to the same area and their mean taken. This seemed on the whole the fairest way to average the results, in view of the fact that owing to differences in shape certain curves would have less weight than others if plotted in terms of equal maxima or equality at any arbitrarily chosen wave-length. According to the results of the paper just previous in this series, the one thing common to all the curves would be their area when the comparison light and the light dispersed into a spectrum were identical.

The data of all observers are given in Table I., with their mean. Table II. gives the mean values in terms of the

Table I.

Normal Spectrum Luminosity Curves of Eighteen Observers.

Observer.	·481	·498	·518	·537	·556	·576	·595	·615	·635	·655 μ
1. P.W.C.	3·4	5·9	13·75	17·4	17·95	17·1	14·0	9·55	5·2	1·7
2. L.H.K.	4·15	7·1	15·5	18·7	18·6	16·5	12·2	7·5	3·65	1·15
3. E.J.E.	3·05	5·9	13·8	18·4	18·95	17·15	13·35	8·65	3·85	1·4
4. W.M.M.	3·6	7·4	15·2	18·45	19·05	16·65	12·45	7·75	3·8	1·15
5. M.D.C.	3·3	6·55	15·0	19·3	19·25	17·6	12·75	7·65	3·45	1·35
6. F.E.C.	2·3	5·05	13·9	17·9	19·1	17·55	14·3	9·4	4·95	1·95
7. M.L.I.	3·25	5·45	12·75	17·8	18·9	17·8	13 7	9 05	4·55	1·9
8. M.L.	3·3	7·7	14·2	18·7	19·8	18·2	12·3	7·3	3·45	1·35
9. H A.R.	3·3	6·9	14·2	17·7	17·8	17·2	13·45	9·0	4·65	2·00
10. L.R.G.	2·65	6·95	15·9	19·55	19·9	17·2	12·3	7·7	2·9	0·9
11. M.M.	3·9	7·2	14·7	17·35	18·5	16·65	13·15	8·05	3·40	1·2
12. C.F.L.	2·7	5·45	13·2	18·1	18·7	17·5	14·15	9·2	4·9	1·6
13. A.J.S.	2·55	5·7	15·2	19·05	19·5	17·45	13·5	8·25	4·1	1·4
14. A.W.	1·6	3·65	12·65	16·9	18·05	18·25	15·6	11·05	5·85	2·3
15. H.H.M.	2·6	6·1	14·85	18·0	19·45	17·8	13·95	8·4	4·15	1·3
16. T.W.R.	3·7	7·4	14·15	19·9	20·1	17·5	12·0	6·7	3·15	1·25
17. H.E.I.	3·0	6·1	15·35	18·75	19·3	17·4	13·45	8·25	3·9	1·5
18. G.C.	2·15	5·2	12·6	18·8	19·4	17·1	14·0	9·3	5·0	1·6
Mean	3·03	6·2	14·55	18·38	19·03	17·47	13·36	8 52	4·16	1·5

Table II.

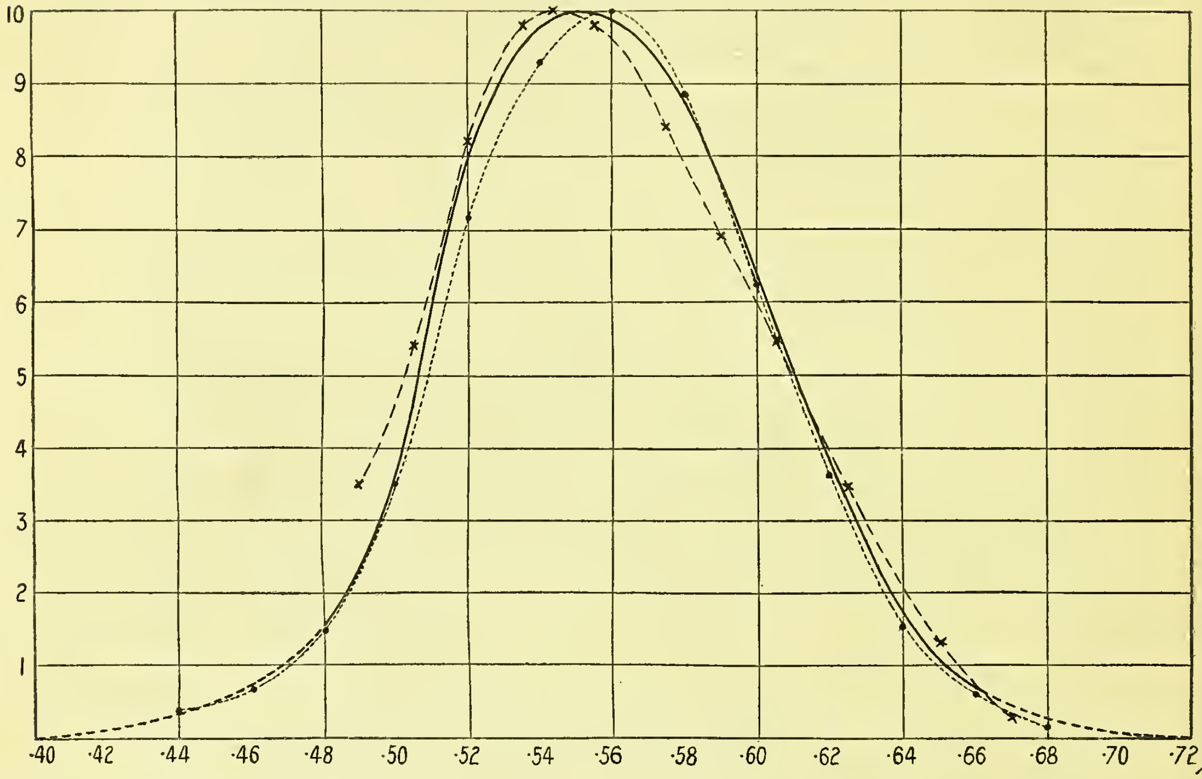
Relative Luminous Efficiencies of Spectral Radiations.

λ.		λ.	
·44 μ	·029 *	·57 μ	·948
·45	·047 *	·58	·875
·46	·073 *	·59	·763
·47	·107 *	·60	·635
·48	·154	·61	·509
·49	·235	·62	·387
·50	·363	·63	·272
·51	·596	·64	·175
·52	·794	·65	·104
·53	·912	·66	·068 *
·54	·977	·67	·044 *
·55	1·000	·68	·026 *
·56	·990		

* Extrapolated.

maximum unity, constituting a table of relative luminous efficiencies of the spectral colours. In fig. 2 is plotted the

Fig. 2.



Average Normal Spectrum Luminosity Curve of Eighteen Observers at 25 metre candles illumination of MgO surface.

Dashed line — . . Kœnig's data, for his own eye, equality of brightness method.
Dotted line . . . . Thürmel's " " flicker method.

mean luminosity curve. Dotted in are the curves obtained by Kœnig *, for his own eye (equality of brightness method), and by Thürmel † (flicker method, also for his own eye). The new curve, as it happens, is about midway in character between the two latter.

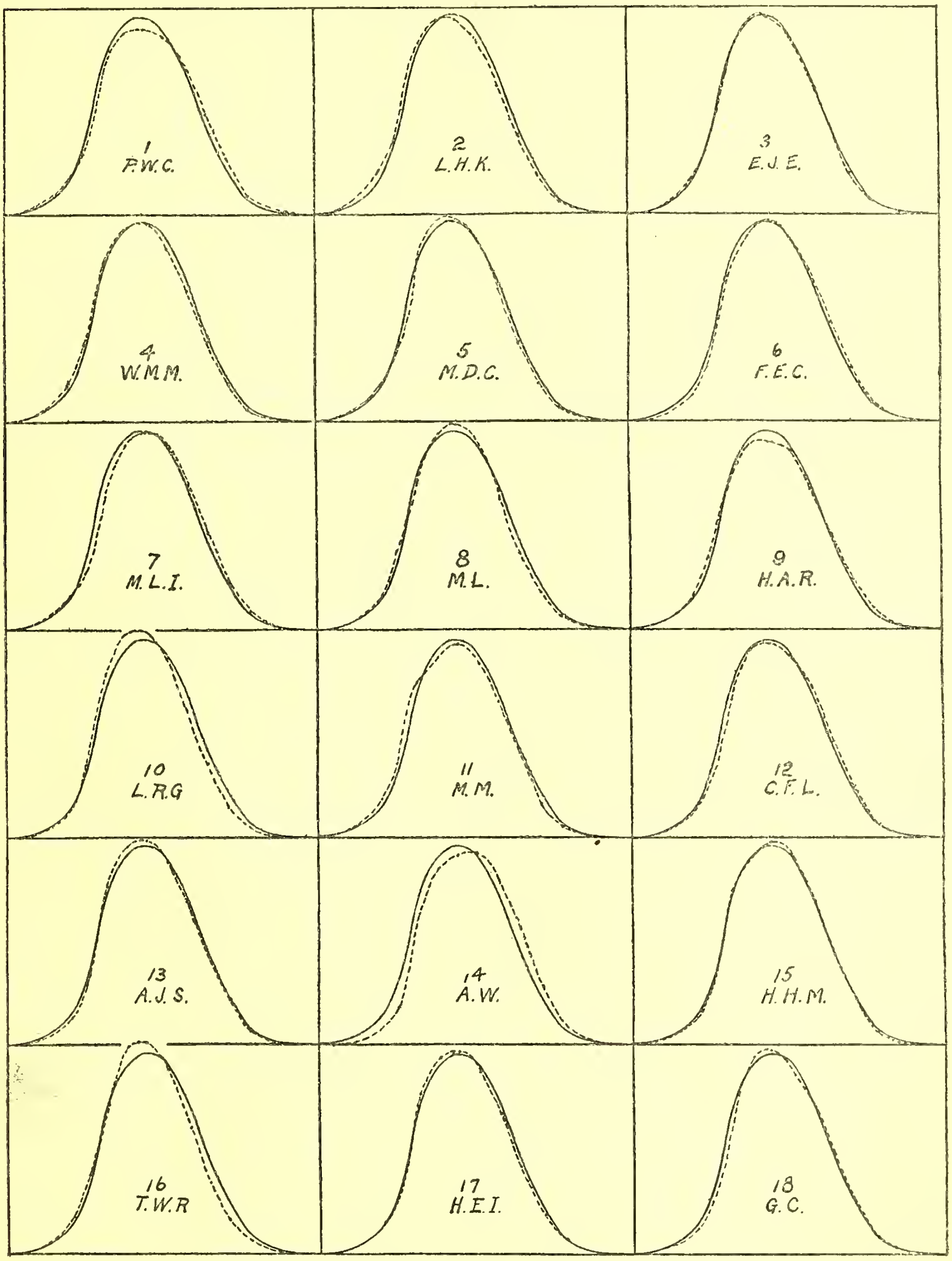
Fig. 3 gives all the eighteen curves, each plotted separately along with the average curve. Study of these shows their range in position and shape. It will be noted that certain of the curves are comparatively broad, others narrow. A rather common difference is in the relative value in the green, *i. e.*, at ·52 μ. Many of the curves would be alike except for their flatness or prominence at this point. These latter differences can probably be ascribed to differences in the pigmentation

* Data for High Illumination as reduced by Nutting, Bull. Bur. of Stds. vol. vii. no. 2, p. 235.

† *Annalen der Physik*, vol. xxxiii. p. 1154 (1910).

of the fovea. There is besides a considerable range in position, although the maxima of all curves lie very nearly together, namely at ·55 μ.

Fig. 3.



Luminosity curves of all observers compared with the average curve.

Discussion.

Use of Luminosity Curve.—To the standard conditions of heterochromatic photometry which have been outlined above must now be added the use of the average eye as defined by the data of Table II. Upon inspection of the curves of fig. 3, it is clear that the individual variations are of such magnitude as to prevent the use of more than a very few eyes for coloured light photometry where any marked colour difference exists. The extreme observers, for instance, would measure a red light against a "white" light with a difference of 100 per cent. These differences become very much less when the light sources under comparison are any of the ordinary illuminants, but some means of obviating them is necessary in order to reach a complete solution of the photometric problem. An approximation to the condition of using a normal eye can of course always be reached by securing the cooperation of a large number of observers, and until each standardizing laboratory is prepared to obtain luminosity curves of its observers and correct their photometric work accordingly, this is the procedure to be recommended.

The most rational manner of approximating to the average eye is, however, by ascertaining the luminosity curve of each observer in the laboratory and calculating the correction to be applied to his results for the energy distribution under measurement. According to the results of this investigation, the total light is accurately the sum of the parts as measured under the standard photometric conditions. It should therefore be possible to apply known energy distributions to the observers' luminosity curves for certain physical conditions of relative intensity (such, for instance, as equality at $\cdot 58\,\mu$), and then from the relative areas of standard and test-lamp luminosity curves in the case of the observers' eyes and in the case of the average eye to obtain the correction which would bring the observers' results to average.

This mode of correction was accordingly tested. A "4 watt" carbon lamp was measured on the spectrophotometer, and by the above method its visible energy distribution was found to correspond to that of a black body at 2080° abs. The normal equal-energy spectral luminosity curves of the two observers which varied most from the normal (14 toward red, 10 toward blue) were then multiplied at each wave-length by the energy values for the 4 watt lamp (BB at 2080°), and for the tungsten lamp at 1·15 w.p.c. (BB at 2360°), the latter values being taken arbitrarily as unity at $\cdot 58\,\mu$. The areas of the four curves, measured by a planimeter, indicated that observer 10 should measure the tungsten lamp against the carbon as 3·3 per cent. higher than observer 14. An actual test, where the two lamps were measured against a comparison lamp of intermediate colour, gave a difference of 2·8 per cent. By this method of correction, therefore, a difference of nearly 3 per cent. in the readings of the two observers is reduced to $\frac{1}{2}$ per cent. As these differences are of the order of magnitude of the errors with the planimeter and photometric methods, the test is taken to show the validity and feasibility of such a method of correction.

A possible and very attractive means of incorporating the average eye into photometric measurement is the development of some sensitive radiometer to supplant the eye. Such an instrument so screened as to respond at the different wave-lengths in accordance with the data of Table II. would, if sufficiently sensitive, offer the most acceptable solution to the problem. Perhaps the photoelectric cell will do this.

Summary.

This paper brings to a close the investigation on coloured light photometry as previously laid out. The practical result of the study has been to arrive at a choice of instrument and of conditions best suited to this kind of photometry, and to establish the characteristics of the average eye by which such measurements are to be carried out. By confining the actual comparison of differently coloured lights to the standardizing laboratory, so that all practical photometry is that of lights of the same colour, a solution of the problem is offered through the use of the results of this investigation. The results of the photometric method here advocated hold for the average eye under the most important illumination conditions. It is inevitable that correction factors will need to be applied to these values whenever the effective illuminations are widely different from that here adopted as standard. But such corrections will apply to the less frequent and less important conditions, that is, to ones where the loss in accuracy involved by the use of more or less rough corrections is of least importance.

Physical Laboratory,
National Electric Lamp Association,
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